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The macroeconomic effects of government debt in a stochastic growth model

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Abstract

This paper studies how government liabilities affect macroeconomic aggregates in a standard general equilibrium growth model. There are two principal results: (i) Though it is often thought that fiscal deficits crowd out investment, this paper shows that deficit-financed cuts in distortionary income taxation may stimulate investment even if agents expect future taxes on capital income to be higher. This result is dependent on the values of two key parameters: the elasticity of labor supply and the degree of persistence in the government debt process. (ii) The economy's response to an increase in government expenditure depends on how it is financed. Distortionary tax finance may lead to a decline in output, consumption, and investment. In contrast, deficit finance may increase output and consumption.

Key words: Government debt; Fiscal policy; Distortionary tax; Ricardian equivalence

JEL classification: E62

1. Introduction

This paper studies how government liabilities affect macroeconomic aggregates. The primary goal is to provide a general equilibrium framework for analyzing how debt-financed changes in fiscal policy affect various series, and how these results in turn depend on key substitution and persistence parameters. The model

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builds in failure of Ricardian equivalence by imposing distortionary income taxation. As a result, changes in the level of government debt influence incentives to work and save and thereby induce shifts in real variables. I attempt to extend the advances in the existing literature on fiscal policy by simultaneously relaxing typical assumptions such as tax finance, nondistortionary taxation, and inelastic labor supply. In doing so, I concentrate on two policy issues:

(i) What is the effect on investment and output of a deficit-financed tax cut? Do higher deficits crowd investment and output out, or do the lower taxes have a stimulatory effect? This inquiry has received prior attention in the literature. Dotsey (1994) demonstrates that the type of taxation plays a decisive role. He shows that existing theory, which predicts an expansion in output and investment in response to deficit-financed cuts in the distortionary tax rate on capital income, rests on the assumption that higher government debt is paid for with lump sum taxation in the future.[1] Instead he establishes, in a model with inelastic labor supply, that lower tax rates paid for by higher deficits lead to *reductions* in investment and output when the deficit is paid for tomorrow by income taxation rather than lump sum taxation. The prospect of future distortionary taxation (a direct consequence of the government's intertemporal budget constraint) lowers the after-tax return to saving, discouraging capital formation.

I follow in the tradition of this analysis by extending the investigation to an environment of elastic labor supply. This paper shows that the degree to which labor is willing to intertemporally substitute hours in response to changes in the after-tax real wage is a principal factor in determining the reaction of investment and output to an increase in government indebtedness. In contrast to the Dotsey result, when labor is elastically supplied and when the same tax rate is levied on labor and capital income, a deficit-financed tax cut may increase investment and output. This prediction does not depend on future taxes being lump sum and occurs even though agents correctly perceive the consequences that higher debt levels have for future distorting tax liabilities.

A deficit-financed tax cut implies that, although rates are lower today, they will be higher tomorrow. The lower rates on labor income motivate a substitution away from leisure and increase output. On the other hand, an increase in future distortionary tax rates means a lower after-tax return to saving, leading to higher consumption. Investment will rise as long as output increases by more than consumption.[2]

The results in this paper demonstrate the importance of elastic labor supply in generating higher investment in response to a deficit-financed tax cut. However, they also indicate that variable hours may not be sufficient for this to

[1] For example, see Abel (1982), Dotsey (1990), and McGrattan (1994).

[2] This is consistent with Dotsey and Mao (1994) who use a different stochastic process for fiscal policy. In that model, states of nature with low labor income tax rates and high debt motivate an increase in labor hours, increasing the marginal productivity of capital and investment demand.

occur. In particular, if government debt is modeled as following a first-order autoregressive process, innovations may instead crowd out investment if they are sufficiently persistent, even when hours are elastically supplied. In previous work, Campbell (1994) and Baxter and King (1993) have emphasized the theoretical relevance of persistence parameters in the tax policy process when the government cannot take on debt. The theory in this paper is based upon (i) exogenous stochastic processes for government expenditure and government debt, (ii) intertemporal elasticity in leisure, and (iii) distortionary income taxation. This provides a rich framework for analyzing, not only the permanence of innovations in government debt, but also the ways in which labor elasticities and persistence parameters interact to influence the impact on output, investment, and consumption.

(ii) How does the way in which government expenditure is financed matter? Previous work has demonstrated the contractionary effect of government consumption when distortionary taxes are levied to pay for it; on the other hand, lump sum finance is known to stimulate labor supply and output due to its negative impact on wealth (e.g., Aiyagari et al., 1992; Barro, 1981, 1987; Baxter and King, 1993; Campbell, 1994; Dotsey and Mao, 1994; Hall, 1980). I use the model here to analyze the impact effects of deficit-financed government spending, and contrast the results with those of tax-financed expenditures. The results here are consistent with previous findings that distortionary tax finance is contractionary. On the other hand, this paper shows that when government spending is paid for by increasing debt, it is possible for labor supply and output to rise in response, depending on parameter values.

The rest of this paper is organized as follows. In the next section, I start by reviewing the stochastic growth environment and the balanced growth relationships it generates. I then briefly describe the solution procedure and calibration assumptions. Section 3 is devoted to discussing the model's predictions concerning the two policy issues outlined above. In each case, I begin by analyzing the outcome for a particular set of benchmark parameters, and then move on to illustrate how the results change as the numerical values of the parameters are modified. Section 4 concludes by summarizing the main findings.

2. A model with elastic labor supply

2.1. The model

Variables at time t, output, technology, and capital are denoted by Y_t, A_t, and K_t. The production function is of the Cobb–Douglas type, explicitly incorporates labor supply, N_t, and assumes technological progress is labor-augmenting:

$$Y_t = (A_t N_t)^{\alpha} K_t^{1-\alpha}. \tag{1}$$

The capital accumulation process is given by

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t - X_t, \tag{2}$$

where δ is the depreciation rate, C_t is consumption, and X_t is government spending. The government debt accumulation equation is given by

$$D_{t+1} = R^g_{t+1}D_t + X_t - \tau_t Y_t, \tag{3}$$

where D_t is government debt at time t, R^g_{t+1} is the gross interest rate on government debt, assumed to be one period certain, and τ_t is the rate of income taxation. The equation states the timing assumption for government expenditures and taxation. It says that they affect the stock of debt with a one-period lag; hence government liabilities are measured as beginning-of-period debt.

In order to maintain constant steady state hours (balanced growth) and separable utility, I use an objective function with log utility for consumption and power utility for leisure:

$$U(C_t, 1-N_t) = \log(C_t) + \theta\frac{(1-N_t)^{1-\gamma_n}}{1-\gamma_n}. \tag{4}$$

Power utility for leisure nests two special cases often analyzed in the real business cycle literature: the divisible labor model of log utility ($\gamma_n = 1$) and the indivisible labor model of Hansen (1985) and Rogerson (1988) where workers choose lotteries over hours ($\gamma_n = 0$). I define $\sigma_n \equiv 1/\gamma_n$ as the elasticity of intertemporal substitution in leisure. A representative agent maximizes the present discounted value of momentary utility functions:

$$\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i U(C_{t+i}, 1-N_{t+i}). \tag{5}$$

The agent maximizes (5) at each date t over consumption, C_t, leisure, $1-N_t$, capital holdings, K_{t+1}, and government debt holdings, D_{t+1}, subject to the following flow of wealth budget constraint:

$$C_t + K_{t+1} + D_{t+1} = (1-\delta)K_t + R^g_{t+1}D_t + (1-\tau)Y_t. \tag{6}$$

The resulting first-order conditions are

$$C_t^{-\gamma} = \beta\mathrm{E}_t\{C_{t+1}^{-\gamma}R_{t+1}\}, \tag{7}$$

$$C_t^{-\gamma} = \beta\mathrm{E}_t\{C_{t+1}^{-\gamma}R^g_{t+1}\}, \tag{8}$$

with a static first-order condition for leisure choice:

$$\theta(1-N_t)^{-\gamma_n} = (1-\tau_t)\alpha\frac{A_t^{\alpha}}{C_t}\left(\frac{K_t}{N_t}\right)^{1-\alpha}, \tag{9}$$

where

$$R_{t+1} \equiv (1-\tau_{t+1})(1-\alpha)\left(\frac{A_{t+1}N_{t+1}}{K_{t+1}}\right)^{\alpha} + (1-\delta), \tag{10}$$

where R is the gross marginal product of capital.[3]

2.2. The balanced growth path

In this section I describe the balanced growth path. This analysis creates a benchmark which can be used to study steady state deviations resulting from stochastic shifts in fiscal policy in the experiments that follow. The driving variable of steady state growth is technology assumed to grow at the constant gross rate $G \equiv A_{t+1}/A_t$. Along the balanced growth path I look for capital, output, consumption, and government debt to grow at this common rate. Then, one can find constant ratios of variables by imposing that this growth rate be consistent with the first-order conditions and accumulation equations. Accordingly, (10) shows that the gross rate of return on capital is a constant R, and along the balanced growth path (7) implies

$$G = \beta R. \tag{11}$$

This can be replaced in (10) to obtain the constant technology to capital ratio,

$$\frac{AN}{K} = \left[\frac{G/\beta - (1-\delta)}{(1-\alpha)(1-\tau)}\right]^{1/\alpha} \approx \left[\frac{r+\delta}{(1-\alpha)(1-\tau)}\right]^{1/\alpha}, \tag{12}$$

where lower-case letters denote logs and the approximate equality obtains from setting $R \approx 1 + r$. This in turn can be used in the production function (1) to obtain the constant output to capital ratio,

$$\frac{Y}{K} \approx \frac{r+\delta}{(1-\alpha)(1-\tau)}. \tag{13}$$

The capital accumulation equation can be used to solve for the constant consumption to capital ratio given by

$$\frac{C}{K} \approx \frac{(r+\delta)(1-X/Y)}{(1-\alpha)(1-\tau)} - (g+\delta). \tag{14}$$

Finally, the steady state relationship between taxes and debt is given by (3):

$$\tau = D/Y(R-G) + X/Y, \tag{15}$$

where I have used the fact that $R^g = R$ in the steady state.

[3] The specification in (10) shows that I have assumed gross capital income taxation. Using net capital income taxation does not significantly alter the results: the only difference is that the decline of the after-tax real return to capital in response to an increase in capital income taxation is slightly less in that case since only undepreciated capital is taxed.

2.3. Solving the model: Calibration and loglinearization

Off the balanced growth path, the model consists of a system of nonlinear expectational difference equations. Following the method of Campbell (1994), these equations can be approximated by loglinearizing them so that the method of undetermined coefficients can be used to solve the system. The various balanced growth path ratios computed above are then used in the loglinearization process to model fluctuations by taking first order Taylor expansions around steady state ratios of variables. To this end, the following parameters are set as in Campbell (1994): the quarterly log technology growth rate, $g = 0.005$, the quarterly log real return on capital, $r = 0.015$, the steady state government spending to output ratio, $X/Y = 0.20$, $\alpha = 0.667$, and $\delta = 0.025$. Further, I set the quarterly debt to output ratio, $D/Y = 2.2$ (or 0.55 at an annual rate). Eq. (15) then implies a steady state value for $\tau = 0.222$ and a government spending to debt ratio, $X/D = 0.09$. I use the benchmark value of $N = \frac{1}{3}$ as the steady state allocation of hours to market activities as advocated by Prescott (1986).

Finally, I assume that the log government expenditure, x_t, and log government debt, d_t, deviate from steady state by following first-order autoregressive processes, leaving the income tax process to be determined by those from (3). In symbols,

$$x_{t+1} = \rho x_t + \varepsilon_{t+1}, \qquad -1 \leq \rho \leq 1,$$

$$d_{t+1} = \phi d_t + u_t, \qquad -1 \leq \phi \leq 1. \tag{16}$$

Both ε_t and u_t are assumed to be white noise random variables. Note that D_{t+1} depends only on variables known at time t, the return on government debt, R^g_{t+1}, government spending, X_t, the tax rate, τ_t, and output, Y_t. It follows that the random shock in the debt process is dated at t.

The solution to the model represents the log of each endogenous variable as an approximate linear function of the log of each state variable, and gives the impact effect of changes in the state variables on output, consumption, and capital formation. From (3) and (16) it is clear that to analyze an increase in debt financed by a drop in current taxation, there must be a positive innovation to u_t holding fixed the level of government spending, x_t, and accrued debt, d_t. Fixing the level of debt inherited from last period, d_t, the consequences of adding to the stock of debt this period are found by varying u_t. To analyze the effects of a deficit-financed tax cut, I allow a positive innovation in u_t, but assume all subsequent shocks to the debt process, u_{t+i} for $i = 1, 2, \ldots$, are zero. The shock u_t then becomes an additional state variable along with the inherited stocks of debt and capital and the current level of government spending. With these four variables, we have everything we need to describe the state of the economy at time t.

Using (16) and with lower-case letters representing logs of variables, the model's solution are denoted as a set of optimal equations for each endogenous variable (the Appendix gives a detailed description of the solution technique):[4]

$$c_t = \eta_{ck}k_t + \eta_{cx}x_t + \eta_{cd}d_t + \eta_{cu}u_t, \tag{17}$$

$$k_{t+1} = \eta_{kk}k_t + \eta_{kx}x_t + \eta_{kd}d_t + \eta_{ku}u_t, \tag{18}$$

$$n_t = \eta_{nk}k_t + \eta_{nx}x_t + \eta_{nd}d_t + \eta_{nu}u_t, \tag{19}$$

$$y_t = \eta_{yk}k_t + \eta_{yx}x_t + \eta_{yd}d_t + \eta_{yu}u_t. \tag{20}$$

The coefficients η_{jk} are the solutions to the model and yield the partial elasticity of the variable j with respect to the variable k, giving the impact effect on the jth variable of a deviation from steady state in the kth variable. In particular, the elasticities with respect to u_t give the impact of a deficit-financed cut in the current tax rate because they show the effect of an increase in debt as measured at the beginning of next period, holding fixed this period's level of government spending, x_t, and the stock of debt already accumulated, d_t (see (3)). The coefficients themselves are complex functions of the calibrated steady state values and are given in the Appendix.

3. Policy experiments

In this section, I consider how innovations to government debt (changes in u_t) and spending (changes in x_t) influence labor supply, output, consumption, and investment. These effects are given by the partial elasticities in the model's loglinear solution (the η's). Before considering the implications of various policy innovations, it may be worthwhile to discuss some properties of the model and how various parameters affect the elasticities.

First, though the log utility specification fixes the intertemporal elasticity in consumption at 1, the absolute values of the labor elasticities in (19) are proportional to σ_n, the intertemporal elasticity in leisure. The more elastic is labor supply as measured by this parameter, the larger these magnitudes will be. This parameter then has obvious implications for the degree with which debt shocks affect the economy since output and consumption are simultaneously determined with hours. Shifts in the after-tax real return to working motivate intertemporal substitution in labor supply. However, it is important to keep in mind that the number of hours devoted to work activities also depends on the level of

[4] In what follows, I ignore technology shocks in order to concentrate on the effects of government debt and spending shocks. Campbell (1994) has already demonstrated that government spending does not have an important effect on the economy's response to technology shocks and vice versa. Similar reasoning can be used here to argue analogously about the addition of government debt shocks.

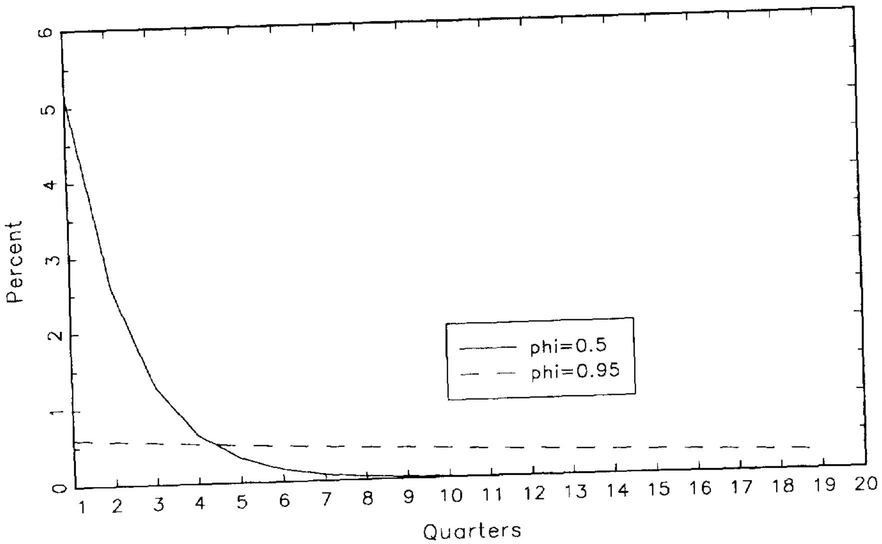

Fig. 1. Path of future taxes after an increase in $u_t, \sigma_n = 1$.

consumption, as well as the after-tax real wage. This can be seen in the static labor–leisure first-order condition (9) where leisure is proportional to the level of consumption. Higher consumption lowers the marginal utility of income and tends to reduce work effort, just as an increase in the real wage works to increase work effort. This is important because debt-induced changes in the gross income tax rate not only alter the after-tax real wage, but also affect the after-tax return to saving and therefore the level of consumption, creating an additional channel of fiscal policy transmission by which labor supply is influenced.

Second, the persistence parameter in the debt process directly affects the persistence of the implied tax process. It may be useful to depict the path of future taxes after a 1% increase in this period's debt (d_{t+1}), when the other variables in the model are held fixed. An increase in debt today, holding fixed government expenditures and the level of debt brought in from last period, means that taxes must adjust downward on impact. However, with the first-order autoregressive process for debt, subsequent debt levels are monotonically decreasing unless debt is a random walk. This implies taxes will have to be somewhat higher next period depending on how persistent the innovation is. Even if the shock is permanent, however, it is an immediate implication of the government's intertemporal budget constraint that future tax rates will still rise. This is because the steady state growth rate of the economy, g, which equals the steady state growth rate of debt, is less than the steady state interest rate paid on the debt. To return to balanced growth with a constant debt to output ratio then requires that a decrease (from steady state) in taxes today must be met with a future increase. Fig. 1 shows the path of future taxes that arise after a temporary, debt-financed, decline in today's rates, for $\phi = 0.5$ and 0.95, holding fixed the growth in output and the interest rate. As the figure shows, more persistent changes in debt bring about more persistent future taxes. When $\phi = 0.5$, taxes have returned to their steady state levels after 8 periods; when $\phi = 0.95$, they still have not returned after 20 periods. In the limit, as ϕ approaches 1, taxes are permanently higher in the new steady state.

Third, elasticities with respect to government expenditure shocks depend on the persistence parameter in the government spending process, ρ, but *not* on the persistence parameter in the government debt process, ϕ. Intuitively, this is because the elasticities are partial, so that η_{nx}, for example, measures the effect of an increase in x_t holding fixed government debt, hence it gives the effect of a tax-financed change in spending. Similarly, the elasticities with respect to government debt depend on ϕ, but not on ρ.

Fourth, an increase in debt, if financed by a reduction in income rather than lump sum taxation, directly alters the labor–leisure tradeoff, but will not produce an income effect which would encourage the agent to work less. This is because the model contains only inside debt: taxes are lower, but only because the agent is lending the government money, an amount that will be returned tomorrow with interest and that will just cover the higher future tax bill.[5] Hence substitution effects dominate the labor–leisure decision in response to debt shocks. This is in contrast to the economy's response to (tax-financed) spending cuts even though both signify a decline in current tax rates. A decrease in wasteful government spending creates a windfall gain in output, so that elasticities with respect to government spending shocks reflect the usual tension between income and substitution effects. As the next sections show, this difference alters the relative magnitudes of debt and spending elasticities and has notable consequences for how the economy responds to deficit-financed spending shocks as compared to tax-financed shocks.

3.1. The effects of deficit financed tax cuts

Elasticities with respect to an increase in u_t give the model's prediction for how a deficit-financed tax cut influences the economy. These elasticities give the effect of an increase in d_{t+1}, holding fixed the other state variables (in particular, the level of government spending, x_t, and the stock of previously accrued debt, d_t) and are denoted $\eta_{\cdot u}$. As a result, the tax process must do the adjusting, and rates fall on impact. To analyze the results, I first focus on the elasticities for a

[5] This is not to deny that the presence of distortionary taxation creates a dead weight loss in wealth. That it does, but the existence of the government's liabilities themselves are not net wealth, so that the only effect variation in government debt has is to distort the decisions of individuals by altering the after-tax returns to working and saving.

Table 1
Elasticities with respect to a deficit financed tax cut with benchmark parameters $\sigma_n = 1$, $\phi = 0.50$

η_{nu}	η_{yu}	η_{cu}	η_{ku}
3.91	2.61	0.53	0.35

The numbers in each cell, left to right, are the elasticities of labor supply, output, consumption, and next period's capital stock with respect to an increase in current debt holding fixed government spending and accumulated debt.

set of benchmark parameters, and then move on to see how the solutions change as the parameter values are varied.

Table 1 gives the numerical values of the partial elasticity of labor hours, output, consumption, and next period's capital stock with respect to a deficit-financed tax cut, when the benchmark is $\sigma_n = 1$ and $\phi = 0.5$ (recall that elasticities with respect to debt innovations are independent of ρ). The coefficient η_{nu} is positive, indicating that a decline in labor income taxation stimulates labor supply. This action increases the after-tax return to working, motivating a substitution away from leisure and into market activities. As a result, output increases as well ($\eta_{yu} > 0$). However, agents rationally expect lower taxes and higher debt today to bring about higher taxes tomorrow in order to satisfy the government's intertemporal budget constraint. This reduces the expected after-tax return to saving and induces a substitution into current consumption ($\eta_{cu} > 0$). Finally, the coefficient η_{ku} gives the effect of a 1% increase in debt on next period's capital stock holding fixed the level of this period's capital stock, thereby showing the impact on investment. As Table 1 illustrates, η_{ku} is positive. A deficit-financed tax cut leads to higher investment even though current consumption rises, because output increases by more than consumption.

Having discussed how the economy responds to these shocks given intermediate persistence in the government's debt process and intermediate degrees of labor elasticity, I now ask how the effects change when the parameters do. Table 2 shows the numerical values of each coefficient for a range of parameter values. The intertemporal elasticity in leisure parameter, σ_n, is set equal to 0, 0.2, 1, 5, and infinity, while ϕ, the persistence parameter on the government debt process, is set equal to 0, 0.50, 0.95, and 1.[6]

Variation in the parameter values generates several important changes in the model's solution. First, inelastic labor supply reverses the sign of η_{ku}. The first column of Table 2 reports the results when the model collapses to the special case of inelastic labor supply, when $\sigma_n = 0$. Though there is now no labor–leisure

[6] Persistence parameters with values of 1 are technically not consistent with balanced growth. However, for the purposes of calibration they can be used to represent the effects in the limit as the shocks become permanent.

Table 2
Consumption, output, employment, and capital elasticities with respect to a deficit-financed tax cut in the variable labor model

	$\sigma_n = 0$	$\sigma_n = 0.2$	$\sigma_n = 1$	$\sigma_n = 5$	$\sigma_n = \infty$
	0.000, 0.000	0.06, 0.70	0.23, 2.73	0.53, 6.52	0.81, 9.99
$\phi = 0.00$	0.000, 0.000	0.10, 1.05	0.40, 4.10	0.95, 9.78	1.46, 15.0
	0.046, 0.000	0.18, 0.70	0.53, 2.61	1.16, 5.84	1.71, 8.56
$\phi = 0.50$	−0.004, 0.000	0.09, 1.05	0.35, 3.91	0.79, 8.76	1.15, 12.8
	0.054, 0.000	0.63, 0.57	1.89, 1.24	3.66, 0.39	5.10, −1.80
$\phi = 0.95$	−0.005, 0.000	0.03, 0.85	0.01, 1.86	−0.28, 0.58	−0.75, −2.71
	0.020, 0.000	1.00, 0.47	2.83, 0.35	5.03, −2.44	6.79, −6.70
$\phi = 1.00$	−0.002, 0.000	−0.02, 0.71	−0.21, 0.53	−0.84, −3.66	−1.66, −10.1

ϕ is the persistence parameter on the government debt process, ϱ is the persistence parameter on the government expenditure process, and σ_n measures elasticity of labor supply. The top two numbers in each cell are η_{cu} and η_{yu}, the elasticities of consumption and output with respect to a deficit-financed tax cut, from left to right. The bottom two numbers in each cell are η_{ku} and η_{nu}, the elasticities of next period's capital stock and this period's employment with respect to a deficit-financed tax cut.

substitution, the incentives to substitute consumption intertemporally are just as they were using the benchmark parameters in Table 1, since this elasticity remains fixed at 1 with log utility over consumption. Lower taxes today continue to create the expectation of higher taxes on capital income tomorrow and discourage saving. As a result, an increase in debt used to finance lower taxation crowds in consumption. This creates an important difference from the benchmark results because it implies that investment and next period's output must fall (capital, which is entirely determined by variables one-period in advance, is now the only variable input in the production function). This is the Dotsey (1994) result, that increases in government debt, when financed by future distortionary taxation, crowd out investment and output. These results indicate the necessity of variable labor supply for output and investment to rise; this is illustrated in Table 2 by the positive values for η_{yu} and η_{ku} when σ_n is positive. The table demonstrates a key distinction between the Dotsey analysis and this one: the willingness of labor to substitute hours intertemporally between leisure and work in response to changes in the after-tax real wage, operates to give the opposite result that output and investment rise in response to a deficit-financed tax cut. Note that the intertemporal elasticity in leisure must be strictly positive, but need not be very large to produce this effect; output and investment both rise in several of the cases in which σ_n is as low as 0.2.

Second, elastic labor supply may not guarantee a rise in investment in response to a deficit-financed tax cut. Notice that the elasticities of labor supply and output are declining in ϕ, showing that a transitory debt shock produces a stronger intertemporal substitution effect. When ϕ is as high as 0.95, elasticities of labor supply and output may even be negative. The reason is that a persistent debt shock

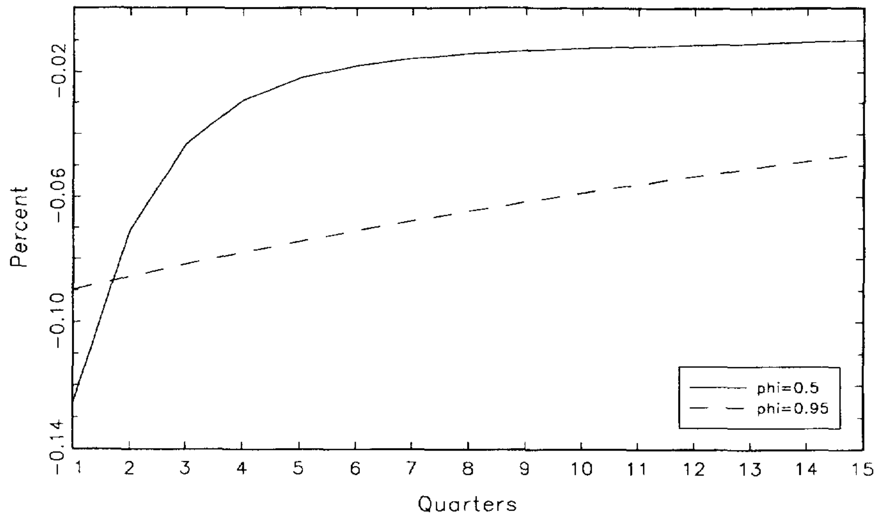

Fig. 2. Response of real rate to expected future taxes, $\sigma_n = 1$

(a tax cut today and increase tomorrow) decreases the expected after-tax return to saving over many future periods, and therefore increases consumption more than a transitory one. The increase in consumption lowers the marginal utility of income and reduces work effort. This phenomenon produces a nonlinearity in the labor elasticities: when the debt shock is transitory, substitution effects are strong, and the response of labor hours to shifts in the after-tax return to working increases as the intertemporal elasticity of leisure, σ_n does. On the other hand, when ϕ is larger (0.95 or 1), the negative impact on the marginal utility of income due to higher consumption dominates the now weaker substitution effect, so that labor and output elasticities fall with σ_n.

Fig. 2 helps to illustrate by showing how the expected after-tax real interest rate responds to higher taxes tomorrow for $\phi = 0.50, 0.95$. Even though the real interest rate declines by more on impact when $\phi = 0.50$ than when $\phi = 0.95$, in every subsequent period the real rate corresponding to greater persistence in the debt process is lower than that associated with more transitory shocks. In addition, the real rate remains significantly below its steady state value for many periods when $\phi = 0.95$, while it returns relatively quickly to its steady state value when $\phi = 0.5$. As a result, a persistent debt shock encourages more dissaving than a transitory one, making η_{cu} large. In summary, consumption elasticities rise with ϕ while labor elasticities fall.

This demonstrates the importance of the degree of persistence in debt changes: as long as an increase in the current deficit is not too persistent, higher debt associated with lower taxes may crowd in investment and output when labor is elastically supplied. On the other hand, persistent debt shocks decrease the expected after-tax return to saving today and in the future, encourage consumption, and reduce the marginal utility of income. The result is a decline in labor supply, output, and investment as expansions in government liabilities become more permanent.

3.2. The effects of a deficit-financed spending increase

In this section, I model the effects of a deficit-financed increase in government consumption (one that is initially financed entirely with debt) and compare it to that of a tax-financed spending increase. Recall that elasticities with respect to x_t give the effects of a tax-financed spending increase because they hold debt (d_t and u_t) constant. With the loglinear approximation, the elasticity of a variable with respect to a deficit-financed spending shock can be represented by a suitable linear combination of the elasticity with respect to an increase in government expenditure holding fixed government debt (a tax-financed spending increase), and the elasticity with respect to an increase in u_t holding fixed government spending (a deficit-financed tax decrease). For example, the elasticity of consumption with respect to a deficit-financed spending shock is simply $\eta_{cx}+X/(GD)\eta_{cu}$, where $X/(GD)$ is the steady state ratio $X_t/D_{t+1} = 0.0896$. That is, the percentage change in consumption for a percent change in purely deficit-financed government spending is simply equal to the percent change in consumption for a percent change in government expenditure, plus the percent change in consumption for a percent change in $D_{t+1}(\eta_{cu})$ times the percent change in debt, D_{t+1}, for a percent change in government expenditure, X_t (equal to $1 \cdot X/GD$ by assumption). One proceeds analogously to compute the elasticities of labor supply, output, and investment with respect to a deficit-financed spending increase.

Note that these elasticities will depend on the persistence parameters of both the government spending and the government debt process. The tax rate (holding fixed output and the interest rate) by definition is not altered initially because spending and debt increase by the same amount (see Eq. (3)). However, as the next tables show, a purely deficit-financed spending increase will have real effects on the economy since consumption and labor supply do not respond by the same absolute amount to innovations in government debt as they do to innovations in government spending.

Table 3 illustrates. The first panel compares labor supply, output, consumption, and capital elasticities with respect to a purely deficit-financed spending increase (first row) and a purely tax-financed spending increase (second row) for the benchmark parameters, $\sigma_n = 1$, $\phi = \rho = 0.50$. The first notable feature is that, with deficit finance, a 1% increase in government expenditure results in an increase in private consumption, whereas tax finance leads to a decrease. In addition, distortionary tax finance decreases labor supply, output, and capital formation on impact (consistent with Baxter and King, 1993; Campbell, 1994), while deficit finance actually increases labor supply and output. The single

Table 3
Comparison of deficit-financed spending increases with tax-financed increases

	η_{ni}	η_{yi}	η_{ci}	η_{ki}
$\sigma_n = 1, \phi = \varrho = 0.50$				
Deficit finance	0.06	0.04	0.02	−0.03
Tax finance	−0.29	−0.19	−0.05	−0.06
$\sigma_n = 1, \phi = 0.95, \varrho = 0.50$				
Deficit finance	−0.12	−0.08	0.12	−0.06
Tax finance	−0.29	−0.19	−0.05	−0.06
$\sigma_n = 0.20, \phi = \varrho = 0.50$				
Deficit finance	0.02	0.01	−0.02	−0.03
Tax finance	−0.08	−0.05	−0.03	−0.04

The first row displays the elasticities of labor supply, output, consumption, and next period's capital stock with respect to a deficit-financed spending increase. The second row gives the same responses with respect to a tax-financed spending increase.

commonality between the two outcomes is that investment declines in both cases.

To understand the differences between tax finance and deficit finance, keep in mind that deficit finance is represented as a linear combination of elasticities with respect to an increase in government expenditure holding fixed debt ($\eta_{\cdot x}$) and elasticities with respect to an increase in debt holding fixed spending ($\eta_{\cdot u}$). The former imposes a tax increase while the latter results in a decline in tax rates. If the positive elasticities with respect to u_t are sufficiently large, they can outweigh the negative elasticities with respect to x_t, so that the net effect on labor supply, output, and consumption is positive. In fact, the absolute magnitudes of the elasticities in Table 1 (showing the effects of a deficit-financed tax decrease) are considerably larger than those in the second row of Table 3 (showing the effects of a tax-financed spending increase), making this outcome possible when the benchmark parameters are used.

The reason for the discrepancy in magnitudes results from a property of the model discussed above: because the debt is owned by the representative agent, deficit-financed changes in taxes have no income effect. Thus, the motivation to substitute into current market activities when a decrease in labor income taxation occurs is not mitigated by an income effect that works to influence labor supply in the opposite direction. On the other hand, higher government spending discourages work effort because it raises the labor income tax, but it also reduces wealth which works to reduce leisure. Though the sign of elasticities with respect to x_t indicate that substitution effects dominate the labor–leisure decision, the absolute magnitudes still reflect the tension between substitution and income effects and so are smaller than those with respect to debt shocks. The net outcome is an increase in consumption, labor supply, and output in response to a deficit-financed spending increase. The growth in nonproductive government expenditure, however, exhausts enough of the economy's resources that the future capital stock falls. Thus, an increase in government consumption may decrease investment no matter how it is financed.

Though this exercise is similar in spirit to that carried out by Dotsey and Mao (1994), it differs in an important way. That paper compares a high spending, low income tax (high debt) shock with a high spending, high income tax (low debt) shock. The result there is analogous to the deficit versus tax finance comparison made here: the former is expansionary and the latter is contractionary. They specify a stochastic process for tax rates and government expenditure so that debt (or lump sum taxation) does the adjusting, and combine higher government spending holding the income tax rate fixed, with lower income tax rates. Both of these actions increase government liabilities. But higher government spending holding fixed distortionary taxation only serves to decrease wealth and motivate a *rise* in work effort and output. Hence in their framework the combination of low income tax rates and high spending is more expansionary than either action in isolation and output increases enough so that, unlike the experiment here, investment also rises.

The key difference is that the exercises themselves are not identical: in their model debt increases by more than what is needed to cover the spending increase, since tax rates move down at the same time spending moves up. This section assumed that government liabilities only rise by the amount needed to cover the spending increase. Note that if, instead of the exercise carried out in this section, the percentage change in government debt for a percent change in government expenditure was sufficiently greater than $1 \cdot X/GD$ (giving more weight to the elasticities which measure the effects of a deficit-financed tax cut), debt would rise by more than what would be necessary to pay for the expenditure shock. This would imply a simultaneous decline in tax rates. In that case, the same result would obtain here as there; in particular, investment could rise along with the other variables. Therefore, the experiments in this section produce predictions about how the economy responds to various combinations of tax and spending innovations that are consistent with the findings of Dotsey and Mao.

I close this section by allowing variation in the benchmark parameters (panels 2 and 3 in Table 3). As the second panel shows, more permanence in the government's debt process (higher ϕ) causes labor supply and output to respond negatively to a deficit-financed spending increase. Again, this comes simply as a result of combining the effects of tax-financed spending and deficit-financed tax cuts. Higher ϕ leads to larger increases in consumption as previously discussed, thereby discouraging work effort as the marginal utility of income falls. Hence when debt shocks are more permanent, deficit-financed government consumption decreases not only investment, as was true of the benchmark parameter results with less persistence in the debt process, but also output. Finally, the third panel shows that decreasing σ_n, the intertemporal elasticity in leisure, and keeping ϕ and ρ fixed at 0.5 returns the labor and output elasticities for deficit finance to positive values, but results in a negative consumption elasticity; in this case more government consumption crowds out private consumption though it does so by less than when the expenditure is financed by taxes. Although the high spending, high debt combination causes increased work effort and higher output, next period's capital stock still falls because output rises by less than government expenditure (net of private consumption).

In summary, the way in which government expenditure is financed can have important effects on the economy. For a benchmark of $\sigma_n = 1$ and $\phi = \rho = 0.5$, distortionary tax finance may lead to a decline in labor hours, output, consumption, and investment, whereas deficit finance may bring about an increase in hours, output, and consumption. However, it is still possible for deficit-financed spending increases to reduce capital formation due to the expansion in wasteful government expenditure.

4. Summary and concluding remarks

In this paper I have explored the consequences of deficit-financed fiscal policy within the context of a fully forward-looking, general equilibrium model. This paper extends the work of Dotsey (1994) who shows that, when labor is inelastically supplied, deficit-financed income tax cuts will reduce investment and output if the debt is paid for by future distortionary taxation. In contrast, the model here predicts that investment and output may respond positively to deficit-financed cuts in income taxation even if the future taxes needed to pay for it are distortionary. There are two key features of the model that drive this result. One is elastic labor supply: a tax cut today and increase tomorrow motivates the agent to consume more *and* work more. It is possible for the increase in work effort and output to outweigh the negative impact on capital formation brought about by a decline in the expected after-tax return to saving. The net result is that output may increase by more than consumption so that investment rises in spite of the fact that agents correctly perceive that higher deficits mean higher taxes on capital income in the future.

The model also shows that elastic labor supply may not be sufficient to guarantee this result. The degree of permanence in government debt shocks also influences how the economy responds to changes in government liabilities. If government debt is modeled as obeying a first-order autoregressive process, sufficiently persistent innovations may crowd out private investment.

Finally, the model shows that impact effects of government expenditure may depend on how the spending is financed. Distortionary tax finance may lead to a decline in output, consumption, and investment. In comparison, the model suggests that for economies where most of the government debt is held inside, deficit finance of higher government expenditures may *increase* output and consumption. The one commonality found between the two forms of finance is that an expansion in wasteful government spending may lead to a reduction in private investment.

A couple of extensions can be explored in future work. It is possible to approximate the Bellman equation using the loglinearization technique in this paper (see Campbell, 1994). One could therefore carry out an extensive analysis of the welfare effects of changes in government debt. Another paper might also explore more comprehensively the dynamic behavior of the economy in response to shifts in government liabilities.

Appendix

This appendix briefly describes the solution technique used to approximate nonlinear equations of the model and gives the resulting elasticities as functions of steady state values. The solution methodology used is that developed by Campbell (1994). I refer the reader to that article for a more complete treatment.

Each evolution equation and first-order equation is approximated as a loglinear function by taking Taylor expansions around steady state values. For example, using the loglinear form of (1), take logs on both sides of (2) and then linearize: divide by K_t

$$\log[\exp(\Delta k_{t+1}) - (1-\delta)] = y_t - k_t + \log[1 - \exp(c_t - y_t) - \exp(x_t - y_t)]. \tag{A.1}$$

On the left-hand side of (A.1) is a nonlinear function $f_1(\Delta k_{t+1}) \equiv \log[\exp(\Delta k_{t+1}) -(1-\delta)]$. This is approximated as $f_1(\Delta k_{t+1}) \approx f_1(g) + f_1'(g)(\Delta k_{t+1} - g)$, where

$$f_1'(g) = \frac{\exp(g)}{\exp(g) - (1-\delta)} \approx \frac{1+g}{\delta+g}. \tag{A.2}$$

On the right-hand side of (A.1) is the nonlinear function

$$q(c_t - y_t, x_t - y_t) \equiv \log[1 - \exp(c_t - y_t) - \exp(x_t - y_t)].$$

This is approximated as

$$q(c_t - y_t, x_t - y_t) \approx q(c-y, x-y) + q_1(c-y, x-y)(c_t - y_t - (c-y)) + q_2(c-y, x-y)(x_t - y_t - (x-y)),$$

where

$$q_1(c-y, x-y) \approx 1 - \frac{(1 - X/Y)(r+\delta)}{(1-\tau)(1-\alpha)(g+\delta)}, \tag{A.3}$$

$$q_2(c-y, x-y) \approx \frac{-X/Y(r+\delta)}{(1-\tau)(1-\alpha)(g+\delta)}. \tag{A.4}$$

Substituting these approximations into (A.1), using the log production function and dropping constants, a loglinear approximate accumulation equation for capital is obtained:

$$k_{t+1} \approx \lambda_1 k_t + \lambda_2(a_t + n_t) + \lambda_3 x_t + \lambda_4 c_t, \tag{A.5}$$

where

$$\lambda_1 \equiv 1 + \frac{r+\delta}{(1+g)(1-\tau)} - \frac{g+\delta}{1+g}, \quad \lambda_2 \equiv \frac{\alpha(r+\delta)}{(1-\tau)(1-\alpha)(1+g)},$$
$$\lambda_3 \equiv \frac{-X/Y(r+\delta)}{(1-\tau)(1-\alpha)(1+g)}, \qquad \lambda_4 \equiv 1 - \lambda_1 - \lambda_2 - \lambda_3. \tag{A.6}$$

The same approach is used to approximate (10). By taking logs of (10), where $d_{t+1} = u_t$, and substituting in (3) for τ_{t+1}, one gets the nonlinear function

$$\begin{aligned} r_{t+1} &= h[r^g_{t+2} + d_{t+1} - y_{t+1}, x_{t+1} - y_{t+1}, d_{t+2} - y_{t+1}, a_{t+1} - k_{t+1}] \\ &\equiv \log[(1-\delta) + [1 - \exp(r^g_{t+2} + d_{t-1} - y_{t+1}) + \exp(x_{t+1} - y_{t+1}) \\ &\quad - \exp(d_{t+2} - y_{t+1})](1-\alpha)\exp(\alpha(a_{t+1} + n_{t+1} - k_{t+1})]. \end{aligned}$$

The approximation of h yields

$$r_{t+1} \approx \xi_1 x_{t+1} + \xi_2(a_{t+1} + n_{t+1}) + \xi_3 k_{t+1} + \xi_4(r^g_{t+2} + d_{t+1}) + \xi_5 d_{t+2}, \tag{A.7}$$

where

$$\xi_1 \equiv \psi_1, \quad \xi_2 \equiv \psi_4 - \alpha(\psi_1 + \psi_2 + \psi_3),$$
$$\xi_3 \equiv \psi_4 + (1-\alpha)(\psi_1 + \psi_2 + \psi_3), \quad \xi_4 \equiv \psi_2, \quad \xi_5 \equiv \psi_3,$$

and where

$$\psi_1 \equiv \frac{-(r+\delta)X/Y}{(1-\tau)(1+r)}, \quad \psi_2 \equiv \frac{-(r+\delta)RD/Y}{(1-\tau)(1+r)}, \quad \psi_3 \equiv \frac{(r+\delta)DG/Y}{(1+r)(1-\tau)},$$
$$\psi_4 \equiv \frac{(r+\delta)\alpha(1 - RD/Y - X/Y + DG/Y)}{(1-\tau)(1+r)}, \quad \psi_4 \equiv \frac{(r+\delta)D/Y}{(1-\tau)(1+r)}.$$

In order to approximate the first-order condition I assume that the variables on the right-hand side of (7) are jointly lognormal and homoskedastic. The first-order condition for optimal consumption choice can then be written in log form as $\mathrm{E}_t \Delta c_{t+1} = \sigma \mathrm{E}_t r_{t+1}$, where lower-case letters denote logs as before. (This uses the formula for the expectation of a lognormal random variable X_{t+1}: $\log(\mathrm{E}_t x_{t+1}) \approx \mathrm{E}_t \log(X_{t+1}) + \frac{1}{2}\mathrm{var}_t \log(X_{t+1})$.) r_{t+1} is the log of the after-tax gross return on capital defined in (10) and is the relevant margin of substitution for consumption between periods t and $t+1$. What is needed for the representative agent's log first-order condition is an approximation of $\mathrm{E}_t r_{t+1}$. Eq. (8) and the assumption of lognormality and homoskedasticity imply that the conditional expected value of $\log R_{t+1}$ is within a constant of the interest rate on the one-period certain government debt, $\log R^g$ at time $t+1$. Hence, focusing on fluctuations I drop the constant and use $\mathrm{E}_{t+1}r_{t+2} = r^g_{t+2}$. Making this substitution in (A.7) leaves us with a first-order linear difference equation for r_t, and along with the loglinear equation for consumption we have a pair of first-order difference equations. Accordingly, to use the method of undetermined coefficients I make simultaneous guesses that the after-tax real interest rate and consumption are functions of each state variable in the model:

$$\begin{aligned} r_{t+1} &= \eta_{rx}x_{t+1} + \eta_{rk}k_{t+1} + \eta_{rd}d_{t+1} + \eta_{ru}u_{t+1}, \\ c_{t+1} &= \eta_{cx}x_{t+1} + \eta_{ck}k_{t+1} + \eta_{cd}d_{t+1} + \eta_{cu}u_{t+1}, \end{aligned} \tag{A.8}$$

where I use the notation η_{yx} to denote the partial elasticity of y with respect to x and where the η coefficients are unknown but assumed to be constant. Finally, and approximation of the labor–leisure first-order condition yields

$$n_t = v[\tilde{\xi}_1 k_t + \tilde{\xi}_2(r^g_{t+1} + d_t) + \tilde{\xi}_3 d_{t+1} + \tilde{\xi}_4 x_t + \tilde{\xi}_5 c_t], \tag{A.9}$$

where

$$v \equiv \frac{(1-N)\sigma_n}{N - (1-N)\sigma_n[((RD + X - GD)\alpha/Y(1-\tau)) - (1-\alpha)]},$$

$$\tilde{\xi}_1 \equiv \frac{(RD + X - GD)(1-\alpha)}{(1-\tau)Y} + 1 - \alpha, \quad \tilde{\xi}_2 \equiv \frac{-RD/Y}{1-\tau},$$

$$\tilde{\xi}_3 \equiv \frac{GD/Y}{1-\tau}, \quad \tilde{\xi}_4 \equiv \frac{-X/Y}{1-\tau}, \quad \tilde{\xi}_5 \equiv -1.$$

Eqs. (A.7), (A.8), and (A.9) along with the loglinear first-order condition for consumption can now be combined and coefficients equated explicitly to obtain the η solutions. The solutions are given below. I start by defining some intermediate parameters:

$$\tilde{\psi}_1 \equiv \frac{-(r+\delta)X/Y}{(1+r)(1-\tau)}, \quad \tilde{\psi}_2 \equiv \frac{-(r+\delta)RD/Y}{(1+r)(1-\tau)}, \quad \tilde{\psi}_3 \equiv \frac{(r+\delta)DG/Y}{(1+r)(1-\tau)}.$$

$$\tilde{\psi}_4 \equiv \frac{(r+\delta)\alpha(1 - RD/Y - X/Y + DG/Y)}{(1+r)(1-\tau)},$$

$$\tilde{\theta}_1 \equiv \tilde{\psi}_1, \quad \tilde{\theta}_2 \equiv \tilde{\psi}_4 - \alpha(\tilde{\psi}_1 + \tilde{\psi}_2 + \tilde{\psi}_3),$$

$$\tilde{\theta}_3 \equiv -(\tilde{\psi}_4 + (1-\alpha)(\tilde{\psi}_1 + \tilde{\psi}_2 + \tilde{\psi}_3)), \quad \tilde{\theta}_4 \equiv \tilde{\psi}_3\phi + \tilde{\phi} + \tilde{\psi}_2, \quad \tilde{\theta}_5 \equiv \tilde{\psi}_2,$$

$$\lambda_1^\star \equiv \lambda_1 + \lambda_2 v\tilde{\xi}_1, \quad \lambda_2^\star \equiv \lambda_3 + \lambda_2 v\tilde{\xi}_4,$$

$$\lambda_3^\star \equiv 1 - \lambda_1 - \lambda_2 - \lambda_3 + \lambda_2 v\tilde{\xi}_5, \quad \lambda_4^\star \equiv \lambda_2 v\tilde{\xi}_2, \quad \lambda_5^\star \equiv \lambda_2 v\tilde{\xi}_3,$$
$$\theta_1^\star \equiv \tilde{\theta}_1 + \tilde{\theta}_2 v\tilde{\xi}_4, \quad \theta_2^\star \equiv \tilde{\theta}_3 + \tilde{\theta}_2 v\tilde{\xi}_1, \quad \theta_3^\star \equiv \tilde{\theta}_4 + \tilde{\theta}_2 v\tilde{\xi}_2 + \tilde{\theta}_2 v\tilde{\xi}_3\phi,$$
$$\theta_4^\star \equiv \tilde{\theta}_5 + \tilde{\theta}_2 v\tilde{\xi}_2, \quad \theta_5^\star \equiv \tilde{\theta}_2 v\tilde{\xi}_5.$$

Solutions for the coefficients in (A.8) are simultaneously determined. η_{rk} solves the quadratic equation, $\eta_{rk}^2 V_2 + \eta_{rk} V_1 + V_0 = 0$, where

$$V_2 \equiv \lambda_4^\star, \quad V_1 \equiv \theta_4^\star(\lambda_1^\star + \lambda_3^\star\eta_{ck}) - \lambda_4^\star\theta_5^\star\eta_{ck} - \theta_2^\star\lambda_4^\star - 1,$$
$$V_0 \equiv \theta_2^\star + \theta_5^\star\eta_{ck}. \tag{A.10}$$

Similarly, η_{ck} solves the quadratic equation $\eta_{ck}^2 Q_2 + \eta_{ck} Q_1 + Q_0 = 0$, where

$$Q_2 \equiv \lambda_3^\star, \quad Q_1 \equiv \lambda_1^\star - 1 + \eta_{rk}(\lambda_4^\star - \lambda_3^\star), \quad Q_0 \equiv -\eta_{rk}\lambda_1^\star. \tag{A.11}$$

Eqs. (A.10) and (A.11) are two nonlinear equations in two unknowns. For each, the economically relevant roots are used in the calculations. In particular, the problem requires a positive root on η_{ck} and a negative root on η_{rk}. Once we have η_{rk} and η_{ck} from (A.10) and (A.11), the other interest and consumption elasticities can be calculated and solved as a linear system of equations (below, I only report the solutions to the elasticities analyzed in the tables and relevant to the excercises in the paper):

$$\eta_{rx} = \frac{(1 - \eta_{rk}\lambda_4^\star)(\theta_1^\star + \theta_5^\star\eta_{cx}) + \theta_4^\star\eta_{rk}(\lambda_2^\star + \lambda_3^\star\eta_{cx})}{1 - \eta_{rk}\lambda_4^\star - \theta_4^\star\rho}, \tag{A.12}$$

$$\eta_{ru} = \frac{(1 - \eta_{rk}\lambda_4^\star)(\theta_3^\star + \theta_5^\star\eta_{cu}) + \theta_4^\star\eta_{rk}(\lambda_5^\star + \lambda_3^\star\eta_{cu})}{1 - \eta_{rk}\lambda_4^\star - \theta_4^\star}, \tag{A.13}$$

$$\eta_{cx} = \frac{(1 - \lambda_4^\star\eta_{rk})\eta_{rx}\rho + (\eta_{rk} - \eta_{ck})(\lambda_2^\star + \lambda_4^\star\eta_{rx}\rho)}{(\rho - 1)(1 - \lambda_4^\star\eta_{rk}) - (\eta_{rk} - \eta_{ck})\lambda_3^\star}, \tag{A.14}$$

$$\eta_{cu} = \frac{(\eta_{rk} - \eta_{ck})(\lambda_4^\star\eta_{ru}\phi + \lambda_5^\star) + \eta_{ru}\phi(1 - \lambda_4^\star\eta_{rk})}{(\phi - 1)(1 - \lambda_4^\star\eta_{rk}) - (\eta_{rk} - \eta_{ck})\lambda_3^\star}. \tag{A.15}$$

The solutions for the income and labor supply elasticities depend on the solutions in (A.10) through (A.15). More intermediate parameters are needed to define these:

$$\omega_1 \equiv \frac{\lambda_1^\star + \lambda_3^\star\eta_{ck}}{1 - \lambda_4^\star\eta_{rk}}, \quad \omega_2 \equiv \frac{\lambda_2^\star + \lambda_3^\star\eta_{cx} + \lambda_4^\star\eta_{rx}\rho}{1 - \lambda_4^\star\eta_{rk}},$$

$$\omega_3 \equiv \frac{\lambda_3^\star\eta_{cu} + \lambda_4^\star\eta_{ru}\phi + \lambda_5^\star}{1 - \lambda_4^\star\eta_{rk}}.$$

With these, the employment and output elasticities are defined (with η_{yu}, for example, denoting the elasticity of output with respect to d_{t+1}) as

$$\eta_{nk} = \nu(\tilde{\xi}_1 + \tilde{\xi}_2\eta_{rk}\omega_1 + \tilde{\xi}_5\eta_{ck}), \quad \eta_{nx} = \nu(\tilde{\xi}_2\eta_{rx}\rho + \tilde{\xi}_2\eta_{rk}\omega_2 + \tilde{\xi}_4 + \tilde{\xi}_5\eta_{cx}),$$

$$\eta_{nu} = \nu(\tilde{\xi}_2\eta_{rk}\omega_3 + \tilde{\xi}_2\eta_{ru}\phi + \tilde{\xi}_3 + \tilde{\xi}_5\eta_{cu}),$$

$$\eta_{yk} = \alpha\eta_{nk} + 1 - \alpha, \quad \eta_{yx} = \alpha\eta_{nx}, \quad \eta_{yu} = \alpha\eta_{nu}.$$

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