Technical Appendix for "Macro Factors in Bond Risk Premia"

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This Technical Appendix contains details relevant to the estimation results in "Macro Factors in Bond Risk Premia."

Small Sample Inference

According to the asymptotic theory for PCA estimation discussed in Section 2, heteroskedasticity and autocorrelation consistent standard errors that are asymptotically $N(0,1)$ can be used to obtain robust t-statistics for the in-sample regressions studied in Section 5.1. Moreover, provided $\sqrt{T}/N$ goes to zero as the sample increases, the $\widehat{F}_t$ can be treated as observed regressors, and the usual t-statistics are valid (Bai and Ng (2006)). To guard against inadequacy of the asymptotic approximation in finite samples, in this section consider bootstrap inference for specifications using four regression models: (i) a model using just the estimated factors in $\overrightarrow{F5}_t$ as predictor variables, (ii) a model using the estimated factors in $\overrightarrow{F5}_t$ and CP_t, (iii) a model using just the single linear combination of five estimated factors, $F5_t$, and (iv) a model using $F5_t$ and CP_t. Small sample inference is especially important when the right-hand-side variables are highly persistent (e.g., Bekaert, Hodrick, and Marshall (1997); Stambaugh (1999); Ferson, Sarkissian, and Simin (2003)) but, as Table 1 demonstrates, none of the factors from our preferred specifications are highly persistent. Nevertheless, we proceed with a bootstrap analysis as a robustness check, by generating bootstrap samples of the exogenous predictors Z_t (here just CP_t), as well as of the estimated factors $\widehat{F}_t$.

Bootstrap samples of $rx_{t+1}^{(n)}$ are obtained in two ways, first by imposing the null hypothesis of no predictability, and second, under the alternative that excess returns are forecastable by the factors and conditioning variables studied above. The use of monthly bond price data to construct continuously compounded annual returns induces an MA(12) error structure in the annual log returns. Thus under the null hypothesis that the expectations hypothesis is true, annual compound returns are forecastable up to an MA(12) error structure, but are not forecastable by other predictor variables or additional moving average terms. Bootstrap sampling that captures the serial dependence of the data is straightforward when, as in this case, there is a parametric model for the dependence under the null hypothesis (Horowitz (2003)). In this event, the bootstrap may be accomplished by drawing random samples from the empirical distribution of the residuals of a $\sqrt{T}$ consistent, asymptotically normal estimator of the parametric model, in our application a twelfth-order moving average process. We use this approach to form bootstrap samples of excess returns under the null. Under the alternative, excess returns still have the MA(12) error structure induced by the use of overlapping data, but estimated factors $\widehat{F}_t$ are presumed to contain additional predictive power for excess returns above and beyond that implied by the moving average error structure.

We take into account the pre-estimation of the factors by re-sampling the $T \times N$ panel of data, x_{it}. This creates bootstrapped samples of the factors themselves. For each i, least squares estimation of $\widehat{e}_{it} = \rho_i \widehat{e}_{it-1} + v_{it}$ yields estimates $\widehat{\rho}_i$ of the persistence of the idiosyncratic errors and of the residuals $\widehat{v}_{it}$, $t = 2, \ldots T$, where recall that $\widehat{e}_{it} = x_{it} - \widehat{\lambda}_i' \widehat{f}_t$. Then $\widehat{v}_{it}$ is re-sampled (while preserving the cross-section correlation structure) to yield bootstrap samples of the idiosyncratic errors $\widehat{e}_{it}$. Bootstrap samples are denoted $\widetilde{e}_{it}$. In turn, bootstrap values of

x_{it} are constructed by adding the bootstrap estimates of the idiosyncratic errors, $\widetilde{e}_{it}$, to $\widehat{\lambda}_i'\widehat{f}_t$. Estimation by the method of principal components on the bootstrapped data then yields a new set of estimated factors. The linear combination $F5_t$ is reestimated in each bootstrap simulation. Together with bootstrap samples of Z_t (also based on an AR(1) model), this delivers a set of bootstrap regressors. Each regression using the bootstrapped data gives new estimates of the regression coefficients in and new $\bar{R}^2$ statistics. This is repeated B times. Bootstrap confidence intervals for the parameter estimates and $\bar{R}^2$ statistics are calculated from $B = 10,000$ replications. The results are reported in Tables 4a-4d for two-, three-, four- and five-year excess bond returns, respectively.

Tables A1-A4 indicate that the results based on bootstrap inference are broadly consistent with those based on asymptotic inference in Tables 2a-2d. Confidence intervals from data generated under the alternative are reported in the columns headed "bootstrap." Confidence intervals from data generated under the null are reported in the columns headed "Bootstrap under the null." The coefficients on the exogenous predictors and estimated factors are all well outside the 95% confidence interval under the no-predictability null. Moreover, the coefficients on factors that are statistically different from zero in Table 2a-2d have confidence intervals under the alternative that exclude zero, indicating statistical significance at the 5 percent level. The exceptions to this are the two inflation factors, which display confidence intervals under the alternative that contain zero for some specifications (as in the asymptotic analysis). However, even these coefficients are too large to be explained under the null of no predictability, and the single linear combination of factors, $F5_t$, is always strongly statistically significant regardless of which excess return is being forecast.

We also compute the small sample distribution of the R^2 statistics. For two-year bond returns, the five-factor model $\overrightarrow{F5}_t$ generates an adjusted R-squared statistic of 22% in historical data; by contrast, using bootstrapped data, the 95% bootstrapped confidence interval for this statistic under the no-predictability null ranges from 1.4% to 1.9%. Similarly, the five factors and CP_t deliver an adjusted R-squared statistic of 45% in historical data; by contrast, using bootstrapped data, the 95% bootstrapped confidence interval for this statistic under the no-predictability null ranges from just 2.3% to 4.3%. The results are similar for bonds of other maturities. In short, the magnitude of predictability found in historical data is too large to be accounted for by sampling error in samples of the size we currently have. The statistical relation of the factors to future returns is evident, even accounting for the small sample distribution of standard test statistics.

Out-of-Sample Inference With Recursively Chosen Factors

In this table, we compare the out of sample forecast errors of a restricted model which includes only a constant with that of an unrestricted model which re-selects the set of forecasting factors $\widehat{F}_t$ in each out-of-sample period. The re-selection procedure works as follows. Let s denote the current out-of-sample period. The set of factors $\{f_t\}_{t=1}^{s}$ is reestimated using principal components on data up to s. The optimal subset of forecasting factors at time s, denoted $\{F_t^s\}_{t=1}^{s-1} \subset \{f_t\}_{t=1}^{s-1}$, is the set that minimizes the BIC in the regression $rx_t^{(n)} = a + X_{t-1}^s b + \epsilon_t$ over all possible subsets that include linear, squared and cubed terms in the factors. Thus,

both the identities and dimension of $\{F_t\}_{t=1}^{s}$ change at each out-of-sample estimation period, s. Note that the observations on F_t^s run from $t = 1$ to $t = s - 1$, since factors are lagged one period to predict excess returns. The optimal set of factors $\{F_t^s\}_{t=1}^{s}$, and corresponding estimates $(\widehat{a}, \widehat{b})$ are used to forecast the $(s+1)$th observation of $rx^{(n)}$.

As above, the out-of-sample performance of the factors is assessed by comparing, MSE_u, the mean-squared forecasting error of the unrestricted model including predictor factors, with MSE_r, the mean-squared forecasting error of the restricted benchmark (null) model based on constant expected returns. The only difference in this case is that the factors are not held fixed throughout the exercise, but are instead chosen in each out-of-sample recursion using the BIC criterion. Otherwise, the procedure here is identical to that described above for assessing the out-of-sample forecasting power of $\overrightarrow{F5}_t$. In the column labeled "MSE_u/MSE_r" of Table A.5, a number less than one indicates that the model with the predictor factors has lower forecast error than a benchmark constant expected returns model.

To assess the statistical significance of the out-of-sample forecasting power of the recursively chosen factors, the column labeled "ENC-t" in Table 3 reports the ENC-t test statistic of Clark and McCracken (2001) for the null hypothesis that the benchmark constant expected excess returns model encompasses the unrestricted model with additional predictors. The alternative is that the unrestricted model contains information that could be used to improve the benchmark model's forecast. "95% BS CV" gives the 95th percentile of the bootstrap distribution of the ENC-t test statistic. The bootstrap procedure assesses the small sample distribution of ENC-t under the null hypothesis that expected excess bond returns are constant. This is implemented by simulating data under the null that the expectations hypothesis holds (see the Appendix for details). In each bootstrap simulation, we simulate data under the null, re-estimate all the factors, go through the BIC factor selection procedure and forecasting exercise, and compute the ENC-t statistic. We then repeat this N times (where $N = 1000$) and tabulate critical values from the set of simulated ENC-t statistics. We use the ENC-t statistic rather than the ENC-NEW statistic here because Clark and McCracken find that the distribution of the ENC-t statistic is less sensitive than is ENC-NEW to variation in the number of forecasting variables over time. Under the null that the benchmark model is true, the ENC-t statistic will be less than or equal to zero. Under the alternative that the additional predictors contain added information, the ENC-t test should be positive. Hence, the ENC-t test is one-sided, and a sufficiently large value of the test statistic indicates a rejection of the null. We calculate the small sample distribution of the ENC-t statistic using a bootstrap under the null of no predictability in excess returns.

For the bootstrap, the new panel of data from which the factors are estimated is created the same way as in tables A.1-A.4, re-sampling from the $T \times N$ panel of data, x_{it}, and estimating the factors by principal components. Estimation by the method of principal components on the bootstrapped data then yields a new set of estimated factors which we use in our BIC optimizing procedure described above.

In each bootstrap simulation we draw randomly with replacement from the residuals of an $AR(12)$ process for the one-year bond yield:

$$y_t^{(1)} = a_0 + a_1 y_{t-1/12}^{(1)} + \cdots + a_{12} y_{t-1}^{(1)} + \epsilon_t \tag{1}$$

We estimate this equation and draw with replacement $\widehat{\epsilon}_t$ to create a new sequence $\{\widetilde{y}_t^{(1)}\}$.

To impose the null of no predictability in excess returns, we create longer holding period returns from the one-year yield imposing the Expectations Hypothesis, which requires that long yields are the average of expected future one-year yields over the life of the bond:

$$y_t^{(n)} = \frac{1}{n} E_t[\sum_{i=1}^{n} y_{t+i-1}^{(1)}]. \tag{2}$$

This is implemented following the recursive procedure in the Appendix to Cochrane and Piazzesi (2005). First we define $x_t = [y_t^{(1)}, y_{t-1/12}^{(1)}, \ldots, y_{t-11/12}^{(1)}]'$, and $e_1 = [1, 0, \ldots, 0]$. Note that $e_1 x_t = y_t^{(1)}$. From this definition of x_t, we can write $x_{t+1/12} = [y_{t+1/12}^{(1)}, y_t^{(1)}, \ldots, y_{t-10/12}^{(1)}]'$ as:

$$x_{t+1/12} = B_0 + B_1 x_t + \Sigma u_{t+1/12}$$

Where: $B_0 = \begin{pmatrix} a_0 \\ 0 \end{pmatrix} B_1 = \begin{pmatrix} a_1 \ldots & a_{12} \\ I_{11x11} & 0_{11x1} \end{pmatrix} \Sigma = \begin{pmatrix} 1 & 0_{1x11} \\ 0_{11x11} & 0_{11x1} \end{pmatrix}$.

So, $E_t[x_{t+1/12}] = B_0 + B_1 x_t$, which we can write more generally as

$$E_t[x_{t+k/12}] = (\sum_{i=0}^{k-1} B_1^i) B_0 + B_1^k x_t$$

by defining $B_1^0 = I_{12x12}$.

Then,

$$E_t[y_{t+k/12}^{(1)}] = e_1((\sum_{i=0}^{k-1} B_1^i) B_0 + B_1^k x_t)$$

Let $k = 12n$. For $n = 1, 2, \ldots, k = 12, 24, \ldots$ we can write this as:

$$E_t[y_{t+n}^{(1)}] = e_1 \left(\left(\sum_{i=0}^{12n-1} B_1^i \right) B_0 + B_1^{12n} x_t \right), n = 1, 2, \ldots \tag{3}$$

If we have $y_t^{(n-1)}$, then, using (2),

$$\begin{aligned} y_t^{(n)} &= \tfrac{1}{n} E_t[\textstyle\sum_{i=1}^{n} y_{t+i-1}^{(1)}] \\ &= \tfrac{1}{n} E_t[\textstyle\sum_{i=1}^{N-1} y_{t+i-1}^{(1)}] + \tfrac{1}{n} E_t[y_{t+n-1}^{(1)}] \\ &= \tfrac{n-1}{n} y_t^{(n-1)} + \tfrac{1}{n} E_t[y_{t+n-1}^{(1)}] \end{aligned}$$

Using our formula (3) above, longer maturity yields can be computed recursively from

$$y_t^{(n)} = \frac{n-1}{n} y_t^{(n-1)} + \frac{1}{n} e_1 \left(\left(\sum_{i=0}^{12(n-1)-1} B_1^i \right) B_0 + B_1^{12(n-1)} x_t \right) \tag{4}$$

Equation (4) is used to compute a new sequence of yields $\{\widetilde{y}_t^{(n)}\}, n = 2, \ldots, 5$, which are then converted to prices by $\widetilde{p}_t^{(n)} = -n\widetilde{y}_t^{(n)}$, and finally to holding period excess returns with $\widetilde{rx}_{t+1}^{(n)} = \widetilde{p}_{t+1}^{(n-1)} - \widetilde{p}_t^{(n)} + \widetilde{p}_t^{(1)}$.

We repeat this procedure 500 times and report the 90th and 95th percentiles of our ENC-t test statistic. Under the null that the restricted (benchmark) constant expected returns model forecast encompasses the model with factors, the ENC-t statistic should be less than or equal to zero. Under the alternative that model with factors adds information, the covariance should be positive. Hence the ENC-t test is one sided.

Table A1: Small Sample Inference, $rx_{t+1}^{(2)}$

Model: $rx_{t+1}^{(2)} = \beta_0 + \beta_1' \overrightarrow{F5}_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	90% CI	95% CI	90% CI
$\widehat{F}_{1t}$	-0.935	(-1.389 -0.474)	(-1.333 -0.538)	(-0.022 -0.020)	(-0.022 -0.020)
$\widehat{F}_{1t}^3$	0.062	(0.023 0.102)	(0.031 0.094)	(0.001 0.001)	(0.001 0.001)
$\widehat{F}_{3t}$	0.177	(-0.043 0.413)	(-0.009 0.371)	(-0.003 0.003)	(-0.003 0.003)
$\widehat{F}_{4t}$	-0.334	(-0.533 -0.137)	(-0.494 -0.182)	(-0.004 0.003)	(-0.003 0.002)
$\widehat{F}_{8t}$	0.352	(0.141 0.542)	(0.184 0.511)	(-0.007 0.008)	(-0.007 0.007)
R^2	0.225	(0.123 0.400)	(0.139 0.381)	(0.014 0.019)	(0.015 0.018)
$\bar{R}^2$	0.217	(0.113 0.393)	(0.130 0.375)	(0.004 0.008)	(0.004 0.008)

Model: $rx_{t+1}^{(2)} = \beta_0 + \beta_1' \overrightarrow{F5}_t + \beta_2 CP_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	90% CI	95% CI	90% CI
$\widehat{F}_{1t}$	-0.745	(-1.141 -0.325)	(-1.075 -0.401)	(-0.025 -0.016)	(-0.024 -0.017)
$\widehat{F}_{1t}^3$	0.055	(0.020 0.091)	(0.026 0.083)	(0.000 0.001)	(0.000 0.001)
$\widehat{F}_{3t}$	0.237	(0.010 0.459)	(0.046 0.412)	(-0.004 0.004)	(-0.003 0.003)
$\widehat{F}_{4t}$	-0.247	(-0.450 -0.055)	(-0.389 -0.099)	(-0.005 0.003)	(-0.004 0.002)
$\widehat{F}_{8t}$	0.244	(0.065 0.424)	(0.095 0.394)	(-0.007 0.008)	(-0.006 0.007)
CP_t	0.395	(0.262 0.519)	(0.283 0.498)	(0.004 0.012)	(0.005 0.011)
R^2	0.455	(0.245 0.548)	(0.268 0.524)	(0.022 0.047)	(0.023 0.043)
$\bar{R}^2$	0.448	(0.235 0.542)	(0.258 0.518)	(0.009 0.034)	(0.010 0.031)

Model: $rx_{t+1}^{(2)} = \beta_0 + \beta_1' F5_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	90% CI	95% CI	90% CI
$F5_t$	0.539	(0.304 0.758)	(0.356 0.729)	(0.008 0.011)	(0.008 0.011)
R^2	0.221	(0.084 0.384)	(0.111 0.368)	(0.008 0.015)	(0.009 0.014)
$\bar{R}^2$	0.219	(0.082 0.383)	(0.109 0.367)	(0.006 0.013)	(0.007 0.012)

Model: $rx_{t+1}^{(2)} = \beta_0 + \beta_1' F5_t + \beta_2 CP_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	90% CI	95% CI	90% CI
$F5_t$	0.427	(0.216 0.626)	(0.252 0.601)	(0.007 0.012)	(0.007 0.012)
CP_t	0.389	(0.255 0.516)	(0.273 0.493)	(0.004 0.011)	(0.005 0.011)
R^2	0.447	(0.215 0.530)	(0.240 0.506)	(0.017 0.041)	(0.019 0.038)
$\bar{R}^2$	0.444	(0.211 0.528)	(0.237 0.504)	(0.013 0.037)	(0.014 0.034)

Notes: See next page.

Notes: Let x_{it} denote the regressor variables used to predict $rx_{t+1}^{(n)}$, including a constant, and let X denote the $T \times K$ vector of such variables, where K is the number of predictors. Let $z_{it}, i = 1, \ldots N, t = 1, \ldots T$ be standardized data from which the factors are extracted. The vector of factors, $\overrightarrow{F5}_t = \left(\widehat{F}_{1t}, \widehat{F}_{1t}^3, \widehat{F}_{2t}, \widehat{F}_{3t}, \widehat{F}_{4t}, \widehat{F}_{8t}\right)' \subset f_t$; $F5_t$ is the single linear combination of these factors formed by regressing the average (across maturity) of excess bond returns on $\overrightarrow{F5}_t$. $\overrightarrow{F5}_t \subset f_t$, where f_t is a $r \times 1$ vector of latent common factors. Denote $\overrightarrow{F5}_t = F_t$. By definition, $z_{it} = \lambda_i' f_t + u_{it}$. Let $\widehat{\lambda}_i$ and $\widehat{f}_t$ be the principal components estimators of λ_i and f_t, and let $\widehat{u}_{it}$ be the estimated idiosyncratic errors. For each $i = 1, \ldots N$, we estimate an AR(1) model $\widehat{u}_{it} = \rho_i \widehat{u}_{it-1} + w_{it}$. Let $\widetilde{u}_{1,.} = u_{1,.}$. For $t = 2, \ldots T$, let $\widetilde{u}_{it} = \widehat{\rho}_i \widetilde{u}_{it-1} + \widetilde{w}_{it}$, where $\widetilde{w}_{i,t}$ is sampled (with replacement) from $\widehat{w}_{.,t}, t = 2, \ldots T$. Then $\widetilde{z}_{it} = \widehat{\lambda}_i' \widehat{f}_t + \widetilde{u}_{it}$. Estimation by principal components on the data $\widetilde{z}$ yields $\widetilde{f}_t$ and therefore $\widetilde{F}_t \subset \widetilde{f}_t$. The remaining regressor, CP_t, is obtained by first estimating an AR(1), and then resampling the residuals of the autoregression. Denote the dependent variable $rx_{t+1}^{(n)}$ as $\widetilde{y}$. Unrestricted samples $\widetilde{y}_t$ are generated as $\widetilde{y} = \widetilde{X}\widehat{\beta} + \widetilde{e}$, where $\widehat{\beta}$ are the least squares estimates reported in column 2, and $\widetilde{e}$ are resampled from least squares MA(12) residuals, and $\widetilde{X}$ is a set of bootstrapped regressors with $\widehat{F}_t$ replaced by $\widetilde{F}_t$. Samples under the null are generated as $\widetilde{y} = \bar{y} + \widetilde{e}^0$, where $\widetilde{e}^0$ is resampled form the residuals of least squares estimated MA(12) process.

Table A2: Small Sample Inference, $rx_{t+1}^{(3)}$

Model: $rx_{t+1}^{(3)} = \beta_0 + \beta_1' \overrightarrow{F5}_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	99% CI	95% CI	99% CI
$\widehat{F}_{1t}$	-1.589	(-2.547 -0.713)	(-2.356 -0.882)	(-0.022 -0.020)	(-0.022 -0.020)
$\widehat{F}_{1t}^3$	0.114	(0.045 0.185)	(0.058 0.173)	(0.001 0.001)	(0.001 0.001)
$\widehat{F}_{3t}$	0.185	(-0.251 0.679)	(-0.175 0.560)	(-0.004 0.003)	(-0.003 0.002)
$\widehat{F}_{4t}$	-0.530	(-0.933 -0.127)	(-0.849 -0.210)	(-0.004 0.003)	(-0.003 0.002)
$\widehat{F}_{8t}$	0.645	(0.259 1.029)	(0.319 0.969)	(-0.008 0.008)	(-0.006 0.008)
R^2	0.189	(0.089 0.377)	(0.103 0.342)	(0.014 0.019)	(0.015 0.018)
$\bar{R}^2$	0.180	(0.079 0.370)	(0.093 0.335)	(0.004 0.008)	(0.004 0.008)

Model: $rx_{t+1}^{(3)} = \beta_0 + \beta_1' \overrightarrow{F5}_t + \beta_2 CP_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	90% CI	95% CI	90% CI
$\widehat{F}_{1t}$	-1.223	(-2.015 -0.431)	(-1.875 -0.545)	(-0.033 -0.021)	(-0.032 -0.022)
$\widehat{F}_{1t}^3$	0.100	(0.035 0.162)	(0.045 0.152)	(0.000 0.002)	(0.001 0.001)
$\widehat{F}_{3t}$	0.300	(-0.132 0.753)	(-0.047 0.667)	(-0.004 0.004)	(-0.004 0.003)
$\widehat{F}_{4t}$	-0.361	(-0.702 0.018)	(-0.632 -0.058)	(-0.005 0.003)	(-0.004 0.002)
$\widehat{F}_{8t}$	0.436	(0.113 0.774)	(0.155 0.718)	(-0.008 0.010)	(-0.007 0.008)
CP_t	0.764	(0.525 0.982)	(0.556 0.941)	(0.005 0.015)	(0.006 0.014)
R^2	0.446	(0.227 0.539)	(0.249 0.522)	(0.021 0.042)	(0.022 0.040)
$\bar{R}^2$	0.439	(0.217 0.533)	(0.239 0.516)	(0.008 0.030)	(0.009 0.028)

Model: $rx_{t+1}^{(3)} = \beta_0 + \beta_1' F5_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	90% CI	95% CI	90% CI
$F5_t$	0.911	(0.473 1.376)	(0.560 1.285)	(0.008 0.011)	(0.008 0.011)
R^2	0.189	(0.055 0.367)	(0.076 0.335)	(0.009 0.015)	(0.009 0.014)
$\bar{R}^2$	0.187	(0.053 0.366)	(0.074 0.334)	(0.006 0.013)	(0.007 0.012)

Model: $rx_{t+1}^{(3)} = \beta_0 + \beta_1' F5_t + \beta_2 CP_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	90% CI	95% CI	90% CI
$F5_t$	0.694	(0.278 1.087)	(0.338 1.026)	(0.007 0.012)	(0.007 0.012)
CP_t	0.754	(0.504 0.971)	(0.546 0.938)	(0.004 0.011)	(0.005 0.010)
R^2	0.442	(0.203 0.521)	(0.226 0.495)	(0.017 0.040)	(0.018 0.037)
$\bar{R}^2$	0.440	(0.199 0.519)	(0.223 0.493)	(0.013 0.035)	(0.014 0.033)

Notes: See Table A1.

Table A3: Small Sample Inference, $rx_{t+1}^{(4)}$

Model: $rx_{t+1}^{(4)} = \beta_0 + \beta_1' \overrightarrow{F5}_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	99% CI	95% CI	99% CI
$\widehat{F}_{1t}$	-2.046	(-3.281 -0.917)	(-3.155 -1.090)	(-0.022 -0.020)	(-0.022 -0.020)
$\widehat{F}_{1t}^3$	0.157	(0.062 0.261)	(0.078 0.240)	(0.001 0.001)	(0.001 0.001)
$\widehat{F}_{3t}$	0.183	(-0.442 0.826)	(-0.293 0.721)	(-0.003 0.003)	(-0.003 0.002)
$\widehat{F}_{4t}$	-0.625	(-1.165 -0.086)	(-1.076 -0.180)	(-0.004 0.003)	(-0.003 0.002)
$\widehat{F}_{8t}$	0.948	(0.433 1.462)	(0.506 1.389)	(-0.007 0.008)	(-0.006 0.007)
R^2	0.167	(0.084 0.357)	(0.098 0.331)	(0.015 0.019)	(0.015 0.018)
$\bar{R}^2$	0.158	(0.074 0.350)	(0.088 0.324)	(0.004 0.008)	(0.004 0.008)

Model: $rx_{t+1}^{(4)} = \beta_0 + \beta_1' \overrightarrow{F5}_t + \beta_2 CP_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	90% CI	95% CI	90% CI
$\widehat{F}_{1t}$	-1.506	(-2.518 -0.440)	(-2.338 -0.640)	(-0.045 -0.029)	(-0.042 -0.030)
$\widehat{F}_{1t}^3$	0.136	(0.052 0.222)	(0.064 0.208)	(0.001 0.002)	(0.001 0.002)
$\widehat{F}_{3t}$	0.353	(-0.215 0.923)	(-0.104 0.805)	(-0.007 0.005)	(-0.006 0.004)
$\widehat{F}_{4t}$	-0.375	(-0.849 0.131)	(-0.754 0.002)	(-0.006 0.004)	(-0.005 0.003)
$\widehat{F}_{8t}$	0.640	(0.166 1.105)	(0.244 1.027)	(-0.008 0.010)	(-0.007 0.008)
CP_t	1.128	(0.789 1.447)	(0.846 1.386)	(0.008 0.019)	(0.008 0.018)
R^2	0.459	(0.254 0.560)	(0.278 0.537)	(0.021 0.041)	(0.022 0.039)
$\bar{R}^2$	0.452	(0.244 0.554)	(0.269 0.530)	(0.008 0.029)	(0.009 0.027)

Model: $rx_{t+1}^{(4)} = \beta_0 + \beta_1' F5_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	90% CI	95% CI	90% CI
$F5_t$	1.188	(0.660 1.784)	(0.735 1.713)	(0.008 0.011)	(0.008 0.011)
R^2	0.167	(0.053 0.343)	(0.071 0.316)	(0.008 0.015)	(0.009 0.014)
$\bar{R}^2$	0.165	(0.051 0.342)	(0.069 0.315)	(0.006 0.013)	(0.007 0.012)

Model: $rx_{t+1}^{(4)} = \beta_0 + \beta_1' F5_t + \beta_2 CP_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	90% CI	95% CI	90% CI
c	0.033	(-1.321 1.274)	(-0.163 0.623)	(0.466 0.479)	(0.467 0.478)
$F5_t$	1.188	(0.660 1.784)	(-1.075 -0.401)	(-0.025 -0.016)	(-0.024 -0.017)
CP_t	0.395	(0.262 0.519)	(0.283 0.498)	(0.004 0.012)	(0.005 0.011)
R^2	0.455	(0.245 0.548)	(0.268 0.524)	(0.022 0.047)	(0.023 0.043)
$\bar{R}^2$	0.448	(0.235 0.542)	(0.258 0.518)	(0.009 0.034)	(0.010 0.031)

Notes: See Table A1.

Table A4: Small Sample Inference, $rx_{t+1}^{(5)}$

Model: $rx_{t+1}^{(5)} = \beta_0 + \beta_1' \overrightarrow{F5}_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	99% CI	95% CI	99% CI
$\widehat{F}_{1t}$	-2.271	(-3.822 -0.735)	(-3.513 -1.023)	(-0.022 -0.020)	(-0.022 -0.020)
$\widehat{F}_{1t}^3$	0.179	(0.056 0.295)	(0.078 0.280)	(0.001 0.001)	(0.001 0.001)
$\widehat{F}_{3t}$	0.182	(-0.612 0.929)	(-0.444 0.790)	(-0.003 0.003)	(-0.003 0.002)
$\widehat{F}_{4t}$	-0.782	(-1.445 -0.125)	(-1.329 -0.269)	(-0.004 0.003)	(-0.003 0.002)
$\widehat{F}_{8t}$	1.129	(0.481 1.841)	(0.598 1.700)	(-0.008 0.008)	(-0.007 0.007)
R^2	0.147	(0.069 0.315)	(0.078 0.294)	(0.014 0.019)	(0.015 0.019)
$\bar{R}^2$	0.138	(0.059 0.308)	(0.068 0.286)	(0.004 0.008)	(0.004 0.008)

Model: $rx_{t+1}^{(5)} = \beta_0 + \beta_1' \overrightarrow{F5}_t + \beta_2 CP_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	90% CI	95% CI	90% CI
$\widehat{F}_{1t}$	-1.629	(-2.914 -0.185)	(-2.638 -0.368)	(-0.049 -0.032)	(-0.047 -0.033)
$\widehat{F}_{1t}^3$	0.154	(0.040 0.264)	(0.057 0.247)	(0.001 0.002)	(0.001 0.002)
$\widehat{F}_{3t}$	0.384	(-0.404 1.112)	(-0.236 0.978)	(-0.007 0.005)	(-0.006 0.004)
$\widehat{F}_{4t}$	-0.485	(-1.116 0.133)	(-1.025 0.017)	(-0.007 0.005)	(-0.006 0.004)
$\widehat{F}_{8t}$	0.764	(0.145 1.351)	(0.242 1.282)	(-0.010 0.012)	(-0.009 0.010)
CP_t	1.341	(0.922 1.711)	(0.993 1.645)	(0.009 0.022)	(0.009 0.021)
R^2	0.421	(0.213 0.514)	(0.242 0.492)	(0.020 0.040)	(0.021 0.038)
$\bar{R}^2$	0.414	(0.203 0.508)	(0.232 0.485)	(0.007 0.028)	(0.008 0.026)

Model: $rx_{t+1}^{(5)} = \beta_0 + \beta_1' F5_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	90% CI	95% CI	90% CI
c	-0.145	(-1.940 1.457)	(-1.725 1.238)	(0.470 0.473)	(0.470 0.472)
$F5_t$	1.362	(0.596 2.087)	(0.756 2.001)	(0.008 0.011)	(0.008 0.011)
R^2	0.146	(0.027 0.303)	(0.046 0.287)	(0.008 0.015)	(0.009 0.014)
$\bar{R}^2$	0.145	(0.025 0.301)	(0.044 0.286)	(0.006 0.013)	(0.007 0.012)

Model: $rx_{t+1}^{(5)} = \beta_0 + \beta_1' F5_t + \beta_2 CP_t + \epsilon_{t+1}$

		Bootstrap		Bootstrap under the null	
x_t	$\widehat{\beta}$	95% CI	90% CI	95% CI	90% CI
$F5_t$	-0.745	(-1.141 -0.325)	(-1.075 -0.401)	(-0.025 -0.016)	(-0.024 -0.017)
CP_t	0.395	(0.262 0.519)	(0.283 0.498)	(0.004 0.012)	(0.005 0.011)
R^2	0.455	(0.245 0.548)	(0.268 0.524)	(0.022 0.047)	(0.023 0.043)
$\bar{R}^2$	0.448	(0.235 0.542)	(0.258 0.518)	(0.009 0.034)	(0.010 0.031)

Notes: See Table A1.

Table A5: Out-of-Sample Predictive Power of Re-Optimized Factors

Row	Forecast Sample	Comparison	MSE_u/MSE_r	ENC-t	90% BS CV	95% BS CV
			$rx_{t+1}^{(2)}$			
1	1985:1-2003:12	*Factors* v.s. *const*	0.853	5.8680	2.529	3.4425
2	1995:1-2003:12	*Factors* v.s. *const*	0.929	4.0447	2.6016	3.2024
			$rx_{t+1}^{(3)}$			
5	1985:1-2003:2	*Factors* v.s. *const*	0.887	5.3203	2.5535	3.4685
6	1995:1-2003:2	*Factors* v.s. *const*	0.894	3.6012	2.6064	3.1744
			$rx_{t+1}^{(4)}$			
9	1985:1-2003:12	*Factors* v.s. *const*	0.908	5.0183	2.5561	3.4866
10	1995:1-2003:12	*Factors* v.s. *const*	0.922	3.3095	2.6032	3.183
			$rx_{t+1}^{(5)}$			
13	1985:1-2003:12	*Factors* v.s. *const*	0.939	4.4705	2.5573	3.4802
14	1995:1-2003:12	*Factors* v.s. *const*	0.940	3.0043	2.6039	3.1785

Notes: See next page.

Notes: Let x_{it} denote the regressor variables used to predict $rx_{t+1}^{(n)}$, including a constant, and let X denote the $T \times K$ vector of such variables, where K is the number of predictors. Let $z_{it}, i = 1, \ldots N, t = 1, \ldots T$ be standardized data from which the factors are extracted. MSE_u is the mean-squared forecasting error of the unrestricted model with factors as predictor variables; MSE_r is the mean-squared forecasting error of the restricted benchmark model that includes only a constant. In the column labeled "MSE_u/MSE_r", a number less than one indicates that the unrestricted model has lower forecast error than the benchmark constant expected returns model. The first row of each panel displays results in which the parameters and factors were estimated recursively, using an initial sample of data from 1964:1 through 1984:12. The forecasting regressions are run for t =1964:1,...,1984:12 (dependent variables from 1964:1-1983:12, independent variable from 1965:1-1984:12), and the values of the regressors at t =1984:12 are used to forecast annual returns for 1975:1-1975:12. All parameters and factors are then reestimated from 1964:1 through 1985:1, and forecasts are recomputed for returns in 1985:2-1986:1, and so on, until the final out-of-sample forecast is made for returns in 2003:12. The same procedure is used to compute results reported in the second row, where the initial estimation period is t =1964:1,...,1994:12. In each forecast period the vector of predictor factors, $\widehat{F}_t \subset \widehat{f}_t$ is chosen according to the BIC criterion, where f_t is a $r \times 1$ vector of latent common factors. The column labeled "Test Statistic" reports the ENC-t test statistic of Clark and McCracken (2001) for the null hypothesis that the benchmark model encompasses the unrestricted model with additional predictors. The alternative is that the unrestricted model contains information that could be used to improve the benchmark model's forecast. "90% bootstrap CV" and "95% BS CV" give the 90th and 95th percentile of the bootstrap distribution of the test statistic. The null is rejected if ENC-t exceeds the critical value. The bootstrap is conducted as follows. By definition, $z_{it} = \lambda_i' f_t + u_{it}$. Let $\widehat{\lambda}_i$ and $\widehat{f}_t$ be the principal components estimators of λ_i and f_t, and let $\widehat{u}_{it}$ be the estimated idiosyncratic errors. For each $i = 1, \ldots N$, we estimate an AR(1) model $\widehat{u}_{it} = \rho_i \widehat{u}_{it-1} + w_{it}$. Let $\widetilde{u}_{1,.} = u_{1,.}$. For $t = 2, \ldots T$, let $\widetilde{u}_{it} = \widehat{\rho}_i \widetilde{u}_{it-1} + \widetilde{w}_{it}$, where $\widetilde{w}_{i,t}$ is sampled (with replacement) from $\widehat{w}_{.,t}, t = 2, \ldots T$. Then $\widetilde{z}_{it} = \widehat{\lambda}_i' \widehat{f}_t + \widetilde{u}_{it}$. Estimation by principal components on the data $\widetilde{z}$ yields $\widetilde{f}_t$ and therefore $\widetilde{F}_t \subset \widetilde{f}_t$. Samples under the null are generated imposing the Expectations Hypothesis starting from an AR(12) for the one-period yield and resampling the residuals from this model. Longer-yields are computed recursively as in the Appendix to Cocharane and Piazzesi (2005). Samples of $rx_{t+1}^{(n)}$ under the null are computed directly from the these longer yields computed under the Expectations Hypothesis.

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