

Appendices to “Expected Returns and Expected Dividend Growth”*

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Appendix A: Derivation of cdy_t

Eq. (4) in the text is based on the derivation in Campbell and Mankiw (1989) for the relation between log consumption and the log of total income flows from aggregate wealth. Campbell and Mankiw move from the consumption-based present-value relation involving future returns (the consumption-wealth ratio) to one involving future income flows. A derivation is given in Campbell and Mankiw (1989) and here.

W_t is total wealth, which consists of N_t shares at time t , each of which have an ex-dividend price, P_t , and dividend payment, I_t :

$$W_t = N_t(P_t + I_t). \quad (\text{A.1})$$

The return on aggregate wealth is defined as

$$R_{t+1} = \frac{P_{t+1} + I_{t+1}}{P_t}. \quad (\text{A.2})$$

Combining (A.1) and (A.2),

$$\frac{W_{t+1}}{N_{t+1}} = R_{t+1} \left(\frac{W_t}{N_t} - I_t \right). \quad (\text{A.3})$$

Eq. (A.3) can be written

$$W_{t+1} = R_{t+1} (N_t + \Delta N_{t+1}) \left(\frac{W_t}{N_t} - I_t \right) ==>$$

$$W_{t+1} = R_{t+1} \left(W_t - I_t N_t + (W_t - I_t N_t) \frac{\Delta N_{t+1}}{N_t} \right)$$

Note that from (A.1), $(W_t - I_t N_t) = N_t P_t$. Thus,

$$W_{t+1} = R_{t+1} (W_t - I_t N_t + P_t \Delta N_{t+1}).$$

The term $P_t \Delta N_{t+1}$ is net new investment, i.e., the net issuance of new shares, ΔN_{t+1} , valued at the ex-dividend price P_t . Investors save by reinvesting a portion of their dividend income in the asset markets.

Eq. (A.3) is of the same form as the accumulation equation for total wealth, $W_{t+1} = R_{t+1} (W_t - C_t)$, and can be linearized in the same way. Campbell and Mankiw do so and derive

$$i_t - w_t = -n_t + E_t \sum_{i=1}^{\infty} \rho^i (r_{t+i} - \Delta i_{t+i}) + \text{constant}, \quad (\text{A.4})$$

where lower case letters denote log variables. Note that i_t in (A.4) is the log per share dividend. To obtain total dividends, I_t must be multiplied by the number of shares N_t ; or in logs we need $i_t + n_t$.

Adding n_t on both sides of (A.4) delivers a present-value relation relating log total dividends to log total wealth:

$$i_t^T - w_t = +E_t \sum_{i=1}^{\infty} \rho^i (r_{t+i} - \Delta i_{t+i}^T + \Delta n_{t+i}) + \text{constant},$$

where i_t^T denotes total (rather than per share) income from aggregate wealth, $i_t^T \equiv i_t + n_t$.

Combining (A.4) with the log-linearized expression for the log consumption wealth ratio

$$c_t - w_t = E_t \sum_{i=1}^{\infty} \rho^i (r_{t+i} - \Delta c_{t+i}), \quad (\text{A.5})$$

yields

$$c_t - i_t^T = E_t \sum_{i=1}^{\infty} \rho^i (\Delta i_{t+i}^T - \Delta n_{t+i} - \Delta c_{t+i}) + \text{constant}. \quad (\text{A.6})$$

This equation is a more general version of Eq. (3.7) in Campbell and Mankiw:

$$c_t - i_t^T = E_t \sum_{i=1}^{\infty} \rho^i (\Delta i_{t+i}^T - \Delta c_{t+i}) + \text{constant}. \quad (\text{A.7})$$

Campbell and Mankiw arrive at Eq. (A.7) by normalizing (in the last step) N_t , the number of shares in each period, to equal one. Although Eq. (A.6) implies that $c_t - i_t^T$ may be related to future changes in the log of the number of shares of asset wealth, this implication is not interesting because the pure number of shares is continuously renormalized by stock splits and reverse splits. Note also that the notation in Campbell and Mankiw (1989) is unfortunate, because in their text, and in their Eq. (3.7), y_t is used to denote log total income (what we denote i_t^T here), whereas in their Appendix, where they derive Eq. (A.7), y_t denotes the log of income per share, i_t .

Eq. (4) is based Campbell and Mankiw's Eq. (A.6), but differs in two respects. First, Campbell and Mankiw assume a particular functional form for investor preferences, and therefore set $E_t \Delta c_{t+i} = \mu + \sigma E_t r_{t+i}$. Second, Eq. (A.6) is expressed in terms of the total income flow from aggregate wealth, i_t^T , whereas in (4), this total is decomposed into its asset wealth and human wealth components using the relation $i_t^T \approx \nu d_t + (1 - \nu) y_t$, where ν is the steady state share of income from asset wealth in total income. Together these assumptions yield the expression

$$cdy_t \equiv c_t - \nu d_t - (1 - \nu) y_t = E_t \sum_{i=1}^{\infty} \rho^i (\nu \Delta d_{t+i} + (1 - \nu) \Delta y_{t+i} - \Delta c_{t+i} - \nu \Delta n_{t+i}). \quad (\text{A.8})$$

For this simple framework, we have assumed that the number of “shares” of human capital are constant, since human wealth is not traded on a stock market. This assumption is inconsequential for the substance of the derivation, since it merely determines whether Δn_{t+i} in Eq. (A.8) is multiplied by the constant ν . Finally, we follow Campbell and Mankiw (1989) and avoid carrying

the term $\nu \Delta n_{t+i}$ around by making an arbitrary normalization that the number shares is always unity. This delivers Eq. (4) in the text.

The steady state income shares ν and $(1 - \nu)$ can be related to the steady state wealth shares ω and $(1 - \omega)$. To see this, assume that the steady state of the economy is characterized by balanced growth at some gross rate $1 + g$, and a constant return on aggregate wealth, $R_{w,t} \equiv R$. These assumptions are standard features of equilibrium growth models. Eq. (A.5) implies that the steady state value of beginning-of-period aggregate wealth is given by

$$\begin{aligned} W_t &= \sum_{i=0}^{\infty} (1 + R)^{-i} C_{t+i} \\ &= \sum_{i=0}^{\infty} (1 + R)^{-i} (I_{t+i} N_{t+i} + P_t \Delta N_{t+1+i}) \\ &= \sum_{i=0}^{\infty} (1 + R)^{-i} (D_{t+i} + P_t \Delta N_{t+1+i}^A + Y_{t+i}), \end{aligned}$$

where N_{t+1}^A denotes the change in the number of shares of asset wealth at time $t + 1$. Using the expression above, and noting that the steady state ratio of aggregate wealth to consumption is given by $(1 + R) / (R - g)$, it is straightforward to show that the steady share of asset wealth in aggregate wealth, ω , is given by

$$\omega = \frac{D_t + (\pi - 1) P_t N_t^A}{D_t + (\pi - 1) P_t N_t^A + Y_t},$$

where D_t , $P_t N_t^A$, and Y_t all grow deterministically at rate $1 + g$, $\pi \equiv (1 + g) / (1 + r - kr + kg)$, and where $k \equiv I_t^T / C_t \geq 1$ is the steady state ratio of total income to total consumption. Notice that when $k = 1$ (there is no saving in steady state), we have $\pi = 1$ and

$$\omega = \frac{D_t}{D_t + Y_t} = \nu,$$

and income shares equal wealth shares.

Appendix B: Data description

The sources and description of each data series we use are listed below.

CONSUMPTION

Consumption is measured as either total personal consumption expenditure or expenditure on nondurables and services, excluding shoes and clothing. The quarterly data are seasonally adjusted at annual rates, in billions of chain-weighted 1996 dollars. The components are chain-weighted together, and this series is scaled up so that the sample mean matches the sample mean of total personal consumption expenditures. Our source is the U.S. Department of Commerce, Bureau of Economic Analysis.

AFTER-TAX LABOR INCOME

Labor income is defined as wages and salaries + transfer payments + other labor income - personal contributions for social insurance - taxes. Taxes are defined as [wages and salaries/(wages and salaries + proprietors' income with IVA and Ccadj + rental income + personal dividends + personal interest income)] times personal tax and nontax payments, where IVA is inventory valuation and Ccadj is capital consumption adjustments. The annual data are in current dollars. Our source is the Bureau of Economic Analysis.

WEALTH

Total wealth is household net worth in billions of current dollars, measured at the end of the period. Stock market wealth includes direct household holdings, mutual fund holdings, holdings of private and public pension plans, personal trusts, and insurance companies. Nonstock wealth includes tangible/real estate wealth, nonstock financial assets (all deposits, open market paper, U.S. Treasuries and Agency securities, municipal securities, corporate and foreign bonds and mortgages), and also includes ownership of privately traded companies in noncorporate equity, and other. Subtracted off are liabilities, including mortgage loans and loans made under home equity lines of credit and secured by junior liens, installment consumer debt, and other. Our source is the Board of Governors of the Federal Reserve System. A complete description of these data may be found at <http://www.federalreserve.gov/releases/Z1/Current/>.

DIVIDENDS

Dividends are constructed from the CRSP index returns. The CRSP dividends, $D_{c,t}$, are scaled by the average ratio of stock market wealth, S_t , to the price of the value-weighted CRSP index, $P_{c,t}$, to reflect dollar values, i.e., $D_t \equiv E(S_t/P_{c,t})D_{c,t}$.

POPULATION

A measure of population is created by dividing real total disposable income by real per capita disposable income. All per capita variables are created by deflating with this measure. Our source is the Bureau of Economic Analysis.

PRICE DEFLATOR

All nominal variables are deflated by the personal consumption expenditure chain-type deflator (1996=100), seasonally adjusted. In principle, the budget constraint implies that one would like a measure of the price deflator for total flow consumption. Since this variable is unobservable, we use the total expenditure deflator as a proxy. Our source is the Bureau of Economic Analysis.

DEFAULT SPREAD, DEF

The default spread is the difference between the BAA corporate bond rate and the AAA corporate bond rate. Our source is the Moody's Corporate Bond Indices.

RELATIVE BILL RATE, *RREL*

The relative bill rate is the three-month Treasury bill yield less its four-quarter moving average. Our source is the Federal Reserve Board.

TERM SPREAD, *TRM*

The term spread is the difference between the ten-year Treasury bond yield and the three-month Treasury bill yield. Our source is the Board of Governors of the Federal Reserve System.

Appendix C: Cointegration tests

This appendix presents the results of cointegration tests. Dickey-Fuller tests for the presence of a unit root in c , y , a , d , and p (not reported) are consistent with the hypothesis of a unit root in those series.

Table C-I reports test statistics corresponding to two cointegration tests. Reported in the far right column are Phillips and Ouliaris (1990) residual-based cointegration test statistics. The table shows both the Dickey-Fuller t-statistic and the relevant 5% and 10% critical values. The test is carried out without a deterministic trend in the static regression. We apply the data-dependent procedure suggested in Campbell and Perron (1991) for choosing the appropriate lag length in an augmented Dickey-Fuller test. This procedure suggests that the appropriate lag length was one for both the $(c, a, y)'$ system and the $(c, d, y)'$ system. The tests reject the null of no cointegration in both systems at the 5% level. The persistent dividend-price ratio displays no evidence favoring cointegration in this sample.

Table C-I also reports the outcome of testing procedures suggested by Johansen(1988, 1991) that allow the researcher to estimate the number of cointegrating relationships. This procedure presumes a p -dimensional vector autoregressive model with k lags, where p corresponds to the number of stochastic variables among which the investigator wishes to test for cointegration. For our application, $p = 3$. The Johansen procedure provides two tests for cointegration: under the null hypothesis, H_0 , that there are exactly r cointegrating relations, the “Trace” statistic supplies a likelihood ratio test of H_0 against the alternative, H_A , that there are p cointegrating relations, where p is the total number of variables in the model. A second approach uses the “L-max” statistic to test the null hypothesis of r cointegrating relations against the alternative of $r + 1$ cointegrating relations.

The critical values obtained using the Johansen approach also depend on the trend characteristics of the data. We present results allowing for linear trends in data, but assuming that the cointegrating relation has only a constant. The articles by Johansen present a more detailed discussion of these trend assumptions. In choosing the appropriate trend model for our data, we are guided by both theoretical considerations and statistical criteria. Theoretical considerations imply

that the long-run equilibrium relation between consumption, labor income, and wealth does not have deterministic trends, although each individual data series may have deterministic trends. The Table also reports the 90% critical values for these statistics.

Both the L-max and Trace test results establish evidence of a cointegrating relation among log consumption, log labor income, and the log of household wealth, and among log consumption, log dividends, and the log of labor income. Table C-I shows that we can reject the null of no cointegration against the alternative of one cointegrating vector. In addition, we cannot reject the null hypothesis of one cointegrating relation against the alternative of two or three.

References

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Table C-I: Cointegration tests

Variables	L-max Test			Trace test			<i>t</i> -test		
	$H_0 : r =$	0	1	2	$H_0 : r =$	0	1	2	$H_0 : \text{no cointegration}$
<i>10% Critical Values</i>		12.10	2.82			13.31	2.71		-2.60
<i>5% Critical Values</i>		14.04	3.96			15.41	3.76		-2.93
<i>d, p</i>		6.06	4.56			10.62	4.56		-0.47
<i>10% Critical Values</i>		18.70	12.10	2.82		26.70	13.31	2.71	-3.52
<i>5% Critical Values</i>		20.78	14.04	3.96		29.68	15.41	3.76	-3.80
<i>c, a, y – 1 lag</i>		25.34	6.57	0.07		31.98	6.64	0.07	-4.13
<i>c, a, y – 2 lags</i>		19.04	8.70	1.58		29.33	10.29	1.58	
<i>c, d, y – 1 lag</i>		27.58	5.36	1.08		34.01	6.43	1.08	-3.77
<i>c, d, y – 2 lags</i>		22.78	3.11	0.72		26.61	3.83	0.72	

Notes: The first two columns report the L-max and trace test statistics described in Johansen (1988) and Johansen (1991). The former tests the null hypothesis that there are r cointegrating relations against the alternative of $r+1$; the latter tests the null of r cointegrating relations against the alternative of p , where p is the number of variables in the cointegrated system. The last column reports the Dickey-Fuller test for $d_t - p_t$ and the Phillips-Ouliaris (1990) cointegration test for (c, a, y) and (c, d, y) . The critical values for the Phillips-Ouliaris tests allow for trends in the data while the Dickey-Fuller regression does not include a trend, since according to the theory, there should be no trend in $d-p$. The variables are consumption c_t , labor income y_t , dividends on the CRSP value-weighted index d_t , price of the CRSP value-weighted index p_t and asset wealth a_t . The null hypothesis is no cointegration; significant statistics at the 10% level are highlighted in boldface. The number of lags in the Johansen tests refers to the VAR specification. The sample is annual and spans the period 1948-2001.